UNIVERSITY OF BATH
DEPARTMENT OF MECHANICAL ENGINEERING

Outline Solution to Examination Question

Examiner: Dr D A S Rees		Date May 2016
Unit Title: Modelling Techniques 2		Unit Code: ME20021
Year: 2015/16	Question Number: 3.	Page 1 of 1

Part		Mark

(a) Separation of variables: let $\theta = T(t)\cos n\pi x$ for $n = 0, 1, 2, \cdots$. Hence $T' = -\alpha n^2\pi^2 T$.

When $n = 0$ we have $T = A$. When $n \neq 0$ we have $T = Ae^{-\alpha n^2\pi^2 t}$.

Reconstructing θ and superposing yields (where 7 marks is allocated if A_0 is missing),

$$\theta = \tfrac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n e^{-\alpha n^2\pi^2 t}\cos n\pi x.$$

Mark: 10

(b) At $t = 0$ we have $\theta = x - x^2$, hence $x - x^2 = \tfrac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos n\pi x$, where

$$A_n = 2\int_0^1 (x - x^2)\cos n\pi x\, dx.$$

After integration we obtain $A_n = 0$ when n is odd, and $A_n = -4/n^2\pi^2$ when n is even. When $n = 0$ we have

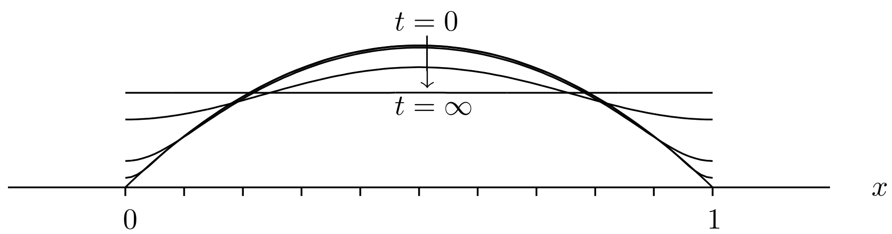
$$A_0 = 2\int_0^1 (x - x^2)\, dx = \tfrac{1}{3}.$$

Hence the full solution is

$$\theta = \tfrac{1}{6} - \sum_{\substack{n=1 \\ n \text{ even}}}^{\infty} 4\frac{\cos n\pi x}{n^2\pi^2} e^{-\alpha n^2\pi^2 t}.$$

Mark: 16

(c) The sketch is as below.



All curves must have zero slope at $x = 0$ and $x = 1$ for full marks.

Mark: 5

(d) The value $\tfrac{1}{2}A_0$ is the mean initial temperature or the long-term temperature which is achieved.

Mark: 2

Total	33

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Part		Mark
(a)	This is bookwork. $\mathcal{F}_s[f''] = \int_0^\infty f'' \sin \omega x \, dx =$ $= \Big[f'\Big]\Big[\sin \omega x\Big]_0^\infty - \Big[f\Big]\Big[\omega \cos \omega x\Big]_0^\infty + \int_0^\infty \Big[f\Big]\Big[-\omega^2 \sin \omega x\Big]\, dx$ $= \omega f(0) - \omega^2 F_s(\omega).$	8
(b)	The application of the Fourier Sine Transform to the wave equation (together with the nonzero boundary condition at $x = 0$) gives $\dfrac{\partial^2 Y_s}{\partial t^2} + c^2\omega^2 Y_s = c^2\omega.$ The general solution is, $Y_s = \dfrac{1}{\omega} + A(\omega)\cos c\omega t + B(\omega)\sin c\omega t.$	6
	At $t = 0$ we have zero displacement, i.e. $y = 0$, and hence $Y_s = 0$. Therefore $A = -1/\omega$.	2
	At $t = 0$ we also have a zero velocity, i.e. $\partial y/\partial t = 0$, hence $\partial Y_s/\partial t = 0$. Therefore $B = 0$.	3
	The solution is $Y_s = \dfrac{(1 - \cos\, c\omega t)}{\omega}$.	2
	Applying the inverse Fourier Sine Transform gives $y = \dfrac{2}{\pi}\displaystyle\int_0^\infty \dfrac{(1 - \cos\, c\omega t)}{\omega}\, \sin\, \omega x\, d\omega.$	2
(c)	The given analytical solution, $y = H(ct - x)$ is equal to 1 when $x < ct$ (i.e. when $ct - x > 0$), and equal to zero when $x > ct$. Therefore the Fourier Sine Transform of this function is $\mathcal{F}_s[H(ct - x)] = \displaystyle\int_0^\infty H(ct - x)\sin \omega x\, dx = \int_0^{ct} 1\, \sin \omega x\, dx = \dfrac{(1 - \cos\, c\omega t)}{\omega}.$ This final answer is the same as Y_s above.	5
	This solution corresponds to a shock wave moving with velocity c in the positive x-direction.	5
	Total	33