Department of Mechanical Engineering, University of Bath

Mathematics 2 ME10305 Sheet 0

The following rather interesting pieces of mathematics were found in this year's Maths 1 examination scripts. The nature of the errors varies substantially from the trivial to the utterly appalling. Determine what the examinees did incorrectly in each case.

Q1. $\dfrac{d\ln|3t|}{dt} = \dfrac{3}{t}$

Q2. $\ln|3t| \Rightarrow \dfrac{3}{3t}$

Q3. $\displaystyle\int \cos^2\theta\, d\theta = \sin^2\theta$

Q4. $\displaystyle\int \cos^2\theta\, d\theta = \tfrac{1}{3}\cos^3\theta$

Q5. $\displaystyle\int \cos^2\theta\, d\theta = \tfrac{1}{3}\sin^3\theta$

Q6. $\displaystyle\int \cos^2\theta\, d\theta = \tfrac{1}{2}\cos^2\theta^2$

Q7. $\displaystyle\int \cos^2\theta\, d\theta = \tfrac{1}{3}\cos^3\theta\sin\theta$

Q8. $x^2 = 3x \Rightarrow x = 3$

Q9. assymtote at $x = 0$

Q10. Three saddles found — no max or min. Must be another point.

Q11. $z = 13e^{j(\theta+2\pi n)}$ with $\theta = 67.38°$

Q12. Sorry for drawing in pen, I forgot I had my pencil with me.

Q13. $\displaystyle\int_0^2 r^3\, dr = \Big[3r^2\Big]_0^2$

Q14. $|-x| < 1 \Rightarrow x < -1$ so there's an infinite radius of convergence

Q15. It is a maxima at $t = 1$. The critical point is a minima.

Q16. $\underline{\mathrm{r}}\cdot\underline{\mathrm{b}} - \underline{\mathrm{a}} = \cdots$

Q17. $2(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}) = 0.0350 + j\,3.190\times10^{-4}$

Q18. $e^x \displaystyle\sum_{n=0}^{\infty} = \frac{x^n}{n!}$

Q19. Using intergration by parts

Q20. $I = \displaystyle\int_0^1 x^2\ln x\, dx.$ Let $x = e^{-y} \implies$
$I = -\displaystyle\int_{e^{-1}}^1 ye^{-3y}\, dy$

Q21. $\displaystyle\int_0^1 x^3\Big[1+9x^4\Big]^{1/2}\, dx = \int_0^1 x^3\Big[1+3x^2\Big]\, dx$

Q22. It is a stationary maximum.

Q23. $72\lambda = 36 \Rightarrow \lambda = \frac{72}{36} = 2$

Q24. Let $x = e^{-y}$ in x^2. Hence $e^{(-y)^2}$ or e^{-y^2}

Q25. $z(x,y) = xy(y+x-3) \implies$
$z(x) = y^2 + 2xy - 3y,$ and $z(y) = x^2 + 2xy - 3x$

Q26. $\dfrac{x^2-2xy-y^2}{(x^2+y^2)^2} = \dfrac{(x-y)^2}{(x^2+y^2)^2}$

Q27. Let $t = x$

Q28. $\displaystyle\int \frac{1}{x^2}\, dx = \ln|x^2|$

Q29. Find $r = 2e^{j\pi/6}$. Let $z = a + bj$ Hence $r = 2$.
So $\sqrt{a^2+b^2} = 2$. Also $\arg(z) = \frac{1}{6}\pi$.
So $\tan\frac{1}{6}\pi = b/a = 1/\sqrt{3} \implies a = \sqrt{3}b$ .....
$a = \sqrt{3}$ and $b = 1$.

D.A.S.R. 02/02/2023