Department of Mechanical Engineering, University of Bath

Mathematics 2 ME10305 Sheet 0

The following rather interesting pieces of mathematics were found in this year's Maths 1 examination scripts. The nature of the errors varies substantially from the trivial to the utterly appalling. The following are in the order in which I saw them. Determine what the examinees did incorrectly in each case.

Q1. $\cos 4\theta + \sin 4\theta = (\cos\theta + \sin\theta)^4$

Q2. $y = e^{-\sin t^2} \quad \Rightarrow \quad y' = -2t\sin t^2 e^{-\sin t^2}$

Q3. $A = 2\pi \int_a^b (y)^2 \sqrt{1 - \left(\frac{dy}{dx}\right)^2}\, dx$

Q4. $t^{-2}e^{2t} \quad \Rightarrow \quad \frac{d}{dt} = e^{2t}(2t^{-2} - 2t^{-3})$

Q5. $\int_0^\pi t^2 \cos t\, dt = \cdots = -2\pi = 2\pi$

Q6. $\int \frac{dt}{2t} = \ln|2t+2| + c$

Q7. Given $y = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2^n}$, then $|x| < \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = 2$.

Hence the radius of convergence is 2.

Q8. $V = \int_0^1 \int_0^2 (x+2y)^2\, dx\, dy = -4/3$

Q9. $\int_0^1 x^3\sqrt{1+9x^4}\, dx = \int_0^1 x^{3/2} + 9x^5\, dx$

Q10. $\int_0^1 x^3\sqrt{1+9x^4}\, dx = \int_0^1 x^3(1+3x^2)\, dx$

Q11. $(x, y) = (0, 3)$ is a straddle point

Q12. $t = 1$ is a minima

Q13. $|\underline{\mathrm{b}}| = \sqrt{6} = 2.45$

Q14. $\int_0^{2\pi} (x\sin x)^2\, dx = \int_0^{2\pi} (x^2 + 2x\sin x + \sin^2 x)\, dx$

Q15. $\int \frac{1}{2t}\, dt = \frac{1}{2}\ln|t| = \ln|t^{1/2}|$

Q16. $\begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0$

Q17. $\lim_{x\to 0}\left(\frac{1-\cos x}{3x^2}\right) = \lim_{x\to 0}\left(\frac{1+\sin x}{6x}\right) = \infty$

Q18. $\frac{t+2}{t^3-t} = \frac{t+2}{t^2(t-1)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t-1}$

Q19. $\int_0^1 \frac{x^2}{\sqrt{1-x^2}}\, dx = \frac{1}{4}\pi + c$

Q20. $u_n = \frac{(-1)^n x^{2n}}{2^n} \Rightarrow u_{n+1} = \frac{(-1)^n x^{2n+1}}{2^n}$

Q21. $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1^n)x^n$

Q22. $y' = \sum_{n=0}^{\infty} (-x)^n \Rightarrow y = c + \sum_{n=0}^{\infty} \frac{(-x)^{n+1}}{n+1}$

Q23. $\int_0^{2\pi} x\sin x\, dx \quad = \quad \Big[(-\cos x)(x) \; - \; (-\sin x)(1)\Big]_0^{2\pi} \quad = \quad -6.136$

Q24. Exam: $V = \int_0^1 \int_0^2 (x+2y)^2\, dx\, dy$

Script: $V = \int_0^1 \int_0^2 x + 2y^2\, dx\, dy$