Feedback on ME10305 Mathematics 2, May 2022

This paper achieved an average of about 54.5% which is substantially lower than what was achieved in recent pre-covid years where the average was usually just above 70% which is regarded as being unacceptably high.

The distribution of marks is close to being uniform. While seven students had a single-figures percentage (the lowest being 2%) and nine were in the teens, 70 were below 40% with 20 in the condonable range (i.e. from 35% to 39%). At the other end of the spectrum, 22 students obtained marks in the 90s with two of those achieving 100%. Nice work.

Given that 72 students obtained first class marks (i.e. at or above 70%) it would seem that the paper could be regarded as being fair. Indeed, the paper itself was deliberately comprised of questions drawn from two previous papers (but with some numbers altered) in order to ensure parity with the pre-covid era. The only exception was question 10 which was made to be longer and more awkward than the rest to try to bring the average below 70%. Ah well.....

As usual, the great majority answered the questions in numerical order.

Q1. ODEs. Average: 6.6/10

Part (a) was generally very well done. Some didn't translate the boundary conditions into the new notation. Many thought that it was an IVP. Some decided to use seven variables instead of the five.

Part (b) was good. Some were fazed by the presence of two logs part of the way through the analysis and that led them astray.

Part (c). Many found the Integrating Factor correctly, but a sizeable minority neglected the minus sign. It was gratifying to see that almost everyone put the equation into the standard form before finding the Integrating Factor.

Q2. Laplace Transforms. Average: 6.0/10.

Parts (a) and (b) formed the bookwork component which should have been free marks for all! Many merely quoted the formula book but no marks accrue from this. The question was phrased as: "Use the above definition of the Laplace Transform to find..." as opposed to "Use the formula book to find...".

Part (c). The partial fractions was straightforward, but only if one realises that the following partial fractions result,

$$\frac{10}{(s+4)(s+9)} = \frac{2}{s+4} + \frac{3}{s+9}$$

may have s replaced by s^2 everywhere to yield,

$$\frac{10}{(s^2+4)(s^2+9)} = \frac{2}{s^2+4} + \frac{3}{s^2+9}.$$

That said, the standard approach with irreducible quadratics also works and is safe even though it takes a little longer.

Q3. Determinants/Gaussian Elimination. Average: 4.9/10.

This split the class roughly into those who got full marks and those who got zero. Some errors were arithmetical, but there seemed to be very many others who either made a cursory start on both parts and then gave up or else made serious algorithmic errors. The overwhelming impression I got was that this just hadn't been revised.

Q4. Least Squares. Average: 5.3/10.

Part (a). The question asked for the theory to be developed rather than to be quoted. Quite a few quoted the correct formula but without derivation and so I couldn't assign any marks. Otherwise this was quite a routine analysis which was done well.

Part (b). This was where most of the marks were lost. Forcing the curve to pass through the origin means that $b = 0$ must be set, and that the equation which is obtained by setting $\partial S/\partial b = 0$ has to be discarded. While many didn't answer this part, some that did either (i) added $(0, 0)$ to the list of points or (ii) changed the first given pair in the question to $(0, 0)$; neither of these strategies forces the curve to pass through the origin.

Q5. Fourier Series. Average: 4.8/10.

Part (a). The sketch was occasionally drawn incorrectly. Incorrect ones almost always had the correct shape for the arch-like function but it should not have dipped below the horizontal axis.

A very large number of people used symmetry arguments to obtain $B_n = 0$ — I was so pleased to see that! The most common error, though, was the retention of t in the Fourier coefficients after supposedly evaluating them.

Part (b). Generally done well. Bizarrely, many didn't write the summation symbol!

Q6. Iteration schemes and root finding. Average: 6.9/10.

This was best question in terms of marks.

Part (a) was almost uniformly perfect. A few students factorised the quadratic into linear factors but then didn't give the roots! Others used the quadratic formula which is fine.

Part (b). Only one set of iterations was needed. Generally very good.

Part (c). Almost everyone did this correctly. Again, only one method needed to be analysed.

Part (d). Again very good in general. Some NR schemes were written down incorrectly and they diverged — this should have been a major hint that something was wrong!

Part (e). Often not completed.

Q7. System of ODEs using eigenvalues. Average: 5.5/10

Part (a). Some found that it took quite a few pages to determine the roots of the cubic for λ. On the other hand, some students found them within a few lines, and the reason was that, when the determinant was expanded, the first line then contains λ only in the form, $\lambda + 3$ or, equivalently, $(-\lambda - 3)$. Thus the cubic takes is immediately of the form of a product of the linear factor $(\lambda + 3)$ and a quadratic in $(\lambda + 3)$. The solution then takes only two more lines.

Part (b). Even if the eigenvalues and/or eigenvectors were incorrect, anyone who attempted to use them to write out the general solution here got the allocated marks.

Part (c). This foxed a few, but it was a nice bonus for those who got parts (a) and (b) right. This part could have been done "by inspection".

Q8. Laplace Transforms. Average 5.8/10.

A technical question. Again I needed to see the derivations of the two transforms given in Part (a). Part (b) was the s-shift theorem — generally well-done. Part (c) was an application of the shift theorem using the results of Parts (a) and (b). I would have preferred to have seen the words, "using the shift theorem" here, rather than just the answer, for it was difficult to know if the formula book had been consulted or not.

Part (d). While convolution is really disliked by a sizeable minority, I was very impressed with the quality of the solutions here. I do not know what it was that I did in the lectures that meant that this was so well-fixed in your minds; in previous year groups such parts of a question are rarely answered well.

Q9. Solution of an inhomogeneous ODE. Average 5.2/10.

Some flew through this in a few lines with very concise writing — nice.

I really thought that this question would have been the best-answered one, the lull before the storm of Q10. This was a standard ODE with a twice repeated λ which almost everyone got right. An amazing number of people thought that $(\lambda+1)^2 = 0$ means that $\lambda = 1, 1$ or that $\lambda = \pm j$. Choosing the latter then really messes up the Particular Integral meaning that no marks could have been given at all. Choosing the former obviously lost some marks, for at least the Particular Integral could be found. This was probably the shortest question on the paper.

Q10. Laplace Transforms for a pair of ODEs. Average 3.4/10,

The least good mark in the exam.

Part (a). This was a result to quote.

Parts (b) and (c). Many obtained the simultaneous equations for $Y(s)$ and $Z(s)$ correctly, but the solutions were often incorrect. Everything was in terms of s^2 but a few stray s^1 terms crept in from somewhere.

When the correct Y and Z expressions were found, there still some donkey work required using partial fractions to obtain expressions that were then easily invertible.

Part (d). $y'(0)$ should have been nonzero despite having applied $y'(0) = 0$ earlier. This was indeed due to the unit impulse forming the forcing function in the y-equation.

So this question was long and had many places where a small arithmetical error would yield some quite horrible-looking expressions. Nevertheless, 20 students answered the question perfectly.

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