Feedback on ME10305 Mathematics 2, May 2016

I endeavoured to make this paper slightly more difficult than last year's paper because the exam average was again deemed to be a little too high. Despite that, this year's exam average came out at about 71.3% and so it will be scaled downwards by 2 or 3 percent at the exam board. There were 14 failures, but that will increase to 16 after scaling. Two students got full marks.

General comments.

Many of the scripts were written quite scruffily, but the maths exam is always a little bit of a marathon and some pace is required. Clearly there will always be a lot of crossings out on some scripts, but that isn't a problem as long I can work out the geography of your solutions easily. There were quite a few scripts where important information had to be searched for, such as answers being placed in a gap two or three lines up from the last thing apparently written.

I don't have an opportunity (given that there is no coursework element for this unit) to air my grievances and extreme annoyance at the frequent use of double negatives. An example is when one wishes to subtract -3 from 4. This should be written as $4 - (-3)$, not as $4 - -3$. The latter, on a subsequent line, often becomes $4 - 3$ and then the analysis which follows is wrong. Quite a few people were guilty of this dangerous and slovenly (sorry, examiner rage...) presentational error.

Q1. ODEs. The average mark was 8.8/10 with 101 students gaining full marks out of the 226 scripts. Everyone attempted the question and no-one obtained zero.

Part (a) was generally very well done. Some didn't translate the boundary conditions into the new notation. Others only quoted the equations for the 3rd and 5th variables, whereas I need all five. Some thought it was of 7th order. Others thought it was an IVP, even though the BCs are given in two different places.

Part (b). Correct almost universally.

Part (c). One person integrated t to get $t^3/3$. But again very very good.

Q2. Laplace Transforms. The average mark was 8.2/10 with 121 students gaining full marks. Three didn't attempt the question but five had zero marks.

Only a small handful of students used the standard method of integration by parts for Parts (a) and (b). Mostly this was done well, but it usually took up more than two pages. By comparison, many students managed to complete this whole question in well under a page.

Part (a) was very very good. The great majority used the "imaginary part of $e^{j\omega t}$" trick. I haven't seen such a large number doing this in the past — I wonder what I did differently in the lectures...

Part (b). Bookwork. Mostly done correctly. A few quoted the formula book, but they got zero marks for that.

Part (c). Some seemed to take a long time to do the partial fractions here. Everything was in terms of s^2, and therefore one didn't need to set

$$\frac{3}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}.$$

Rather, one can do

$$\frac{3}{(s^2+1)(s^2+4)} = \frac{A}{s^2+1} + \frac{C}{s^2+4}$$

because there is no way that a solitary s^1 can appear.

Q3. Matrices/Gaussian Elimination. The average mark was 8.2/10 with 167 students gaining full marks. Everyone attempted the question but 16 had zero marks.

Mostly done very well. I have to admit that a typo had made it through to the exam paper, which is why the solution might have seemed a little unusual, but most got this right and many checked that the solution was correct by multiplying it by the matrix and comparing that with the right hand side vector.

The intended right hand side was $(4, 3, 8, 13)$, for which the solution is $(2, 0, 1, 6)$.

Q4. Simple probability. The average mark was 9.3/10 with 167 students gaining full marks. Everyone attempted the question but two had zero marks.

Part (a). Very few people drew what I would regard as being the best table of data for the information given. There was no need at all to draw the 8×8 grid where all of the 64 entries were $\frac{1}{64}$. I needed the (one dimensional) table of probabilities associated with each potential score obtained. That said, the required probabilities were usually found correctly.

Part (b). Also done well with the most frequent mistake involving the incorrect test of independence.

Q5. Least Squares. The average mark was 7.9/10 with 118 students gaining full marks. Nine didn't attempt the question and nine others had zero marks.

The average mark was unusually high compared with previous years (when it was the very last topic taught, which could be the reason...)

However, some quoted the formula, and therefore marks were lost. Some even quoted the formula for a straight line and used that, and therefore even more marks were lost. In the last part, the aim was to suppress the equation corresponding to the minimisation of S with respect to the constant term (see the outline solutions — this is the lower line of the matrix/vector equation at the top), but some suppressed the other equation.

Q6. Laplace Transforms. The average mark was 6.3/10 with 75 students gaining full marks. 17 didn't attempt the question and 29 had zero marks.

This was quite a technical and austere question, and so I was pleasantly surprised at the quality of the workings with many students. In these cases I saw something very much like how I have written my outline solutions.

Q7. Numerical Mathematics. The average mark was 6.8/10 with 50 students gaining full marks. 12 didn't attempt the question and five had zero marks.

A new topic for this year and therefore I made this queston the easier and shorter one of the two on this topic.

Part (a). I was intrigued by how many students derived the iteration scheme incorrectly (having quoted the correct general formula for Newton-Raphson), and didn't go back to check what had been done wrongly. In all of those cases the mistake was the following,

$$\begin{aligned} x_{n+1} &= x_n - \frac{(x_n^2 - 4)}{2x_n} \\ &= x_n - \frac{x_n^2}{2x_n} - \frac{4}{2x_n} && \text{2 minuses making a minus} \\ &= \frac{x_n}{2} - \frac{2}{x_n}. \end{aligned}$$

Part (b). Not so well done at all. The chief error was neglecting the ϵ^2 term in the numerator, or neglecting an ϵ in the denominator. Newton-Raphson usually converges quadratically, and therefore the ϵ^2 term must be retained.

Q8. Numerical mathematics. The average mark was 4.9/10 with 36 students gaining full marks. 35 didn't attempt the question and 22 had zero marks.

Part (a). A host of different errors crept in here. One was the assumption that h was equal to $1/N$, rather than $2/N$ because the integral was from $x = 1$ to $x = 3$. Another frequent one saw the index of each gridpoint being used instead of the corresponding x=values. Thus instead of having

$$I_4 = \frac{1}{2}\Big[\frac{1}{2}\sqrt{1} + \sqrt{1.5} + \sqrt{2} + \sqrt{2.5} + \frac{1}{2}\sqrt{3}\Big],$$

there was

$$I_4 = \frac{1}{2}\Big[\frac{1}{2}\sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} + \frac{1}{2}\sqrt{4}\Big],$$

and for eight intervals we had

$$I_8 = \frac{1}{4}\Big[\frac{1}{2}\sqrt{0} + \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7} + \frac{1}{2}\sqrt{8}\Big].$$

Even when the correct values of I_4 and I_8 were found, often the exact error wasn't calculatd from the analytical solution, and sometimes the Richardson Extrapolation wasn't performed.

Part (b). Bookwork. Mostly done well.

Part (c). Not bad. The chief error was weighting the R.E. formula towards the less accurate value. The formula for the ratio test gave $2^\alpha = 8.075$. This indicates that $\alpha = 3$, a third order method, and therefore one uses $\alpha = 3$ in the Richardson Extrapolation formula. Some used $\log_2 8.075$ and one even used 8.075!

Q9. Matrices/Eigenvalues/ODEs. The average mark was 7.0/10 with 102 students gaining full marks. Two didn't attempt the question and 12 had zero marks.

Part (a). Wow did this cause a huge amount of algebra for some! I made this one a relatively easy one to do, but it is only easy if one retains all those $(\lambda + 3)$ factors without multiplying out; see the outline solutions online. Two measly marks for two pages of detailed algebra doesn't seem right, and it isn't! If you look at the outline solutions, you'll see how to do this in no more than four lines.

That said, once the correct eigenvalues were found the eigenvectors were almost always determined correctly.

Part (b). I was generous here. In most cases of incorrect answers to Part (a), if those incorrect answers were translated correctly into the appropriate form required for this one-line solution, then I gave the marks.

Part (c). This bit seems complicated, but was quite straightforward. Quite a few students got the correct solution 'by inspection'. Nice one....

Q10. Probability. The average mark was 6.2/10 with 32 students gaining full marks. 11 didn't attempt the question and 17 had zero marks.

Part (a). The question asked for the probability transition matrix to be given. Many didn't, only to have to do it later! Generally this was done well.

Part (b). Also very well done. Those who hadn't written down the matrix used a tree diagram, and I didn't penalise those who then got the required answer.

Part (c). Not well answered in general. This part only attracted three marks, and I should have weighted it a little higher. While the eigenvalues and eigenvectors were often found correctly, many then wrote down a 'solution' in the same way as was asked for in Q9b, which makes no sense in the probability context! Rather, the arbitrary constant multiplying the $\lambda = 1$ eigenvector must be such that the entries add up to 1 — this is because the two outcomes are the only possible things to happen. We may ignore the eigenvalue and eigenvector.

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