Feedback on ME10305 Mathematics 2, May/June 2021

This was a 24 hour open-book examination unlike last year's exam for which the students had three weeks! Last year's average mark was 87.6% which is well above the usual level which generally hovers around 70%. Therefore I strove to reduce the average mark by providing a fairly lengthy Q9, and a Q10 which was very different from anything that was in the lectures or the problem sheets. I was only partially successful because the average mark was just below 80.0%.

Other stats are as follows: the standard deviation was 15.3%, three people were below 40% and ten people gained full marks. Unfortunately one person submitted their thermodynamics script.

As things stand today, June 4th, I am guessing that the Board of Examiners will moderate the unit by subtracting 10%, and should this happen then a further eight students will be below 40%.

Some general points.

In a standard two-hour exam I expect to see scruffy scripts because that's the nature of the beast, but I was very surprised to see so many crossings-out in an open book exam. One student actually scanned a full page of crossings-out!

By way of contrast there was a huge number of very tidy scripts where great care had been taken to make my life easy during the marking — very many thanks for this was appreciated greatly. I would like to single out candidates 11758, 11977, 12293 and 12836, whoever they are, as being exemplars of presentation.

I did have some scripts which were laid out in a double-column format. If I had been marking paper scripts then this would have been fine. But when marking online it is bad enough viewing a portrait page on a landscape screen, and the presence of two columns (three, in one case!), slowed things up quite a bit because a lot of scrolling up and down is required.

There were a few questions which involved finding the values of 'arbitrary' constants to complete an ODE solution. There were maybe 40 instances where these constants were found but the final answers were not quoted on the script! Given that there were also instances where correct values of the constants had been found but where the final answer was incorrect (yes, that happens quite a lot), I really really do need to see the final answer in order to give full marks. An additional bit of class is to stick a box around that final answer, but this isn't essential.

All the question averages are high (apart from Q10) and so I'll just concentrate mainly on the sort of things that went wrong. Generally, I was very pleased indeed with how the questions were tackled.

Q1. ODEs. Average: 9.1/10, standard deviation: 1.4.

Parts (a) and (b). This was a sixth order system of ODEs and it should have been reduced to six 1st order ODEs. Generally answered very well indeed. Two students pointed out that I had given only five boundary conditions....oops on my part, although that didn't stop the question itself being answered perfectly.

Part (c). Very very good generally. Some left the solution in the form $\ln|y| = \cdots$; it would have been better to do the classical $y = \cdots$, for y is the solution not $\ln|y|$.

Part (d). Aaargh, I had a typo on the exam paper which meant that the constant of integration came out to be a bit strange. Didn't seem to affect anyone; the great majority found it easily.

Q2. Laplace Transforms. Average: 8.9/10, standard deviation: 1.9.

Part (a) asked you to write out some results. Everyone got 2 marks.

Part (b). Yes, everyone found the linear factor. Easy marks.

Part (c). This was an extensive analysis. One student wrote the following on his/her script: *A nice bit of s-shift application there. Well-designed question!*. Oh how I laughed for this was my third typo, and it was this typo which resulted in such a long question. My original model solution was for a different initial condition for which the analysis was shorter and easier. So while I am embarrassed, I have to say that you guys hardly noticed, at least in terms of how well this question was done. I am very impressed.

Q3. Determinants/Gaussian Elimination. Average: 9.3/10, standard deviation: 1.8.

Hardly any errors. When you see the outline solutions, bear in mind that there were quite a few ways to achieve the determinant, so your way may possibly not be the same as the one which I used.

I did wonder why some didn't use the row/column manipulation method to evaluate the determinant. In one case the analysis went on for two pages, as opposed to roughly three or four lines.

The average mark for the question would have been even higher if Gaussian Elimination had been used by all. Some decided to use the Gauss-Jordan method. In an exam it is not a good idea not to follow the instruction of the examiner.

Q4. Fourier Series. Average: 6.9/10, standard deviation: 2.7.

Part (a). Some just drew a single straight line; this should have been a sawtooth shape.

Part (b). This was a sine series but a few gave me an A_0 term as well — oops! I frequently use symmetry to make integrations easier, but this idea was misunderstood by some. The sketch of the function, $f(t)$, tells us that it is odd, and hence A_0 and all the other A_n-values are zero. However, some wrote the following

$$A_0 = 2\int_{-\frac{1}{2}}^{\frac{1}{2}} f(t)\,dt = 2\int_0^{\frac{1}{2}} (t - \frac{1}{2})\,dt = \cdots .$$

Strictly, the first integral is correct when we use the $f(t)$ which is shown in the sketch. However, one would need to use $f(t) = t + \frac{1}{2}$ in the range, $-\frac{1}{2} < t < 0$, in order to get correct integral. The second integral is incorrect.

On the same topic, one student wrote: *Upon further inspection, I should have seen that $f(t)$ is odd and saved all that useless integration. Might as well leave it as it is.* Well, it was good integration practice and you got it right that the A-values are zero. If your integration had been incorrect with nonzero A-values, then I would have had to deduct a mark or two because then the proffered answer would be wrong.

Lots of excellent integrations by parts.

Part (c). Some people used $(4 - n\pi)$ as the extra term in the denominator. This is erroneous in two ways. It should have been $(4 - 4n^2\pi^2)$ because the ODE is of second order and the sine is $\sin 2n\pi t$.

Often the summation was missing for the Particular Integral.

Part (d). Pretty good.

General comments: note that $\cos 2n\pi = 1$ and $\sin 2n\pi = 0$ for all integer values of n; sometimes these expressions were left unevaluated or incorrectly evaluated. Note also that $(-1)^{2n+1} = -1$.

Q5. Least Squares. Average: 8.8/10, standard deviation: 2.5.

Part (a). A few decided on deriving the 3×3 matrix corresponding to a full quadratic fit $(ax^2 + bx + c)$ including the constant. Although this wasn't what I asked for, when the extra row and column were removed then resulting formula is fine. However, some people evaluated the full quadratic and merely quoted the a and b values whilst ignoring the nonzero c-value — this is incorrect.

I also asked for the derivation. Just quoting the formula gained no marks.

Part (b) was often correct, but somehow the final solution in part (c) wasn't. In some cases the errors were wildly out for no reason that I could understand, but in others it was clear that numbers were rounded during the calculations and this always introduces errors.

I generally prefer decimal fractions to be used rather than vulgar fractions. Solutions are easier to understand when presented as decimals.

Q6. Iteration schemes and root finding. Average: 8.4/10, standard deviation: 1.9.

Generally extremely good. The most common error was: $2-(1+\epsilon)=1+\epsilon$.

Some errors were introduced by using too few DPs.

Some presented the sketch with a vertical line along $x=1$; this wasn't adequate. There were three or four different sketches which could have been provided. One involved the function itself using your skills from last semester. But there were various ways of drawing two different curves to convince one that there was only one intersection.

There was a third possible iteration scheme: $x_{n+1}=2/(x_n^2+1)$ and its perturbation analysis yields,

$$x_n = 1+\epsilon \quad \Rightarrow \quad x_{n+1}=1-\epsilon+2\epsilon^3.$$

This is a particularly awkward one to analyse and so I am going to add it to next year's problem sheet! Basically, it says that the error changes sign (ϵ in x_n to $-\epsilon$ in x_{n+1}) but successive iterations move just very very slightly closer to the root by an amount of magnitude ϵ^3. So it is worse than ad hoc schemes with a double root, but it is as bad as for a triple root.

Q7. ODE solutions with eigenvectors. Average: 9.1/10, standard deviation: 2.1.

Impressively done! All bar two students found the eigenvalues, although I have to admit that I wouldn't have given you this one as part of a closed-book two-hour exam.

Q8. Laplace Transforms and ODEs. Average 7.7/10, standard deviation: 3.7.

A large standard deviation here because many didn't have a good time whilst others walked it.

By far the quickest way through this question involved solving for X and Y using matrices. With smallish writing this could be done in half a page. See my outline solutions.

Very pleased to see people arguing mathematically, i.e. along the lines of, *the equations for X and Y remain the same when X and Y are swapped, and therefore $X=Y$*. This too shortens the time needed.

One student expressed doubt that $x=y=\sin t$ could be correct, and I am always happy to see people thinking about whether their solution makes physical sense. If one had solved this using eigenvalues/eigenvectors, then one eigenvector would be $(1,1)$, which represents the present solution, while the other is $(1,-1)$. The initial condition was precisely of the form of the first eigenvector, and the subsequent motion continues to reflect that relationship.

Just a heads up that the following partial fractions,

$$\frac{s^2+9}{(s^2+1)(s^2+9)}=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+9},$$

isn't needed because (s^2+9) cancels between the denominator and the numerator. One student took two pages to find that $A=0$, $B=1$, $C=0$ and $D=0$. Quite a few did it this way.

Q9. Some classic but lengthy ODEs. Average 8.1/10, standard deviation: 2.6.

Not much to say here. I did notice that two scripts attempted these ODEs using Laplace Transform methods. This is possible, quite definitely, but it is a bit of a nightmare. However, the scripts made identical errors, which is curious. One of these was to have $\mathcal{L}[3]=3$ as opposed to $\mathcal{L}[3]=3/s$. The workings were precisely the same and the phraseology too. And also for other questions....

I would prefer $(1+t)\sin 2t$ to $\sin 2t\,(1+t)$. The former is unambiguous.

Q10. Least squares with integration. Average: 3.7/10, standard deviation: 4.2.

This was by far the lowest mark for a question and I had intended that it should be so. However, 44 students obtained perfect marks while 120 obtained a zero, which explains the huge standard deviation. I am impressed by the 44.

This was a question where itegrations are used as opposed to summations. The formulae for the unknown constants decouple and they are essentially identical to those for two terms in a Fourier Series. One may extend this question to cover a full Fourier Series and essentially this would tell us that a Fourier Series actually corresponds to least squares fit.

I asked the question, "Are the formulae for a and b familiar? If so, then what do they remind you of?" I had the following suggestions:

They remind me of my childhood.

Ran out of time `∩´ (haha).

They are familiar but it is a secret .... sorry!

BAO, these are cool functions.

Yes.

I won't lie, I have no clue whats going on.

I am not sure what is happening, but I enjoyed the paper.

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