Department of Mechanical Engineering, University of Bath

Mathematics ME10305

EXAM NOTES

I will give details of my office hours in a separate communication, but if you feel that your query is short enough to be described clearly and concisely in an email, and that the answer is likely to be the short, then do please email. I will endeavour to reply within 24 hours. Occasionally I may reply to the whole class if I feel that it is of interest to the whole class to do so, although I will keep anonymous the name of the person who emailed me.

In previous years I have had the occasional student who would send me scans of their working on a more than daily basis and I just don't have the time to find their arithmetical errors. This is because I also have to deal with the second years and, for the final years, I am committed to reading and marking their final year project reports, preparing for their vivas and conducting those vivas. So in the first instance do please check things against the solutions that I have posted online, for both the problem sheets and the past exams. As a student I always found that working with a peer group was very beneficial and I am convinced that that pushed me up a degree classification.

You will have the formula book in the exam and be provided with the NEW university calculators. A link to this is on the Maths 2 web site.

ODEs

When reducing an ODE (or a system of ODEs) to first order form, do not forget to do the same to the boundary conditions.

For first order linear equations, do remember how to find the integrating factor; you are not expected to derive the formula for the integrating factor.

It is possible that I might give you a system of linear, constant coefficient equations to solve. I didn't cover this topic in the ODEs part of the unit, but it was touched upon in Laplace Transforms and in the Matrices sections. If I do ask for the solution of such systems then do follow the method I specify if I specify a method. Otherwise, it might be done either using Laplace Transforms or by adopting a matrix eigenvalue approach. Either method will work, although it is generally safer (i.e. quicker) to go via eigenvalues and eigenvectors.

You will need to remember the formula for the Integrating Factor for 1st order linear equations.

Laplace Transforms

If I ask you to find, say, the Laplace Transform of $\sin \omega t$, then this does not mean finding it in the formula book! I expect a derivation involving an integration!

You will be asked to find the transform of a derivative and one or two functions. I will quote the definition of the Laplace Transform on the paper itself, and I will also quote the definition of certain LT results on the paper if required (e.g. symmetry theorem, convolution theorem), but others (such as the shift theorems) should be learned because I might ask for the derivation.

Unit impulses and how to use them in integrals (such as the LT of an impulse) need to be memorised.

Matrices

Gaussian Elimination always results in an upper triangular matrix. You may interchange rows in order to make arithmetic easier, although it won't really be necessary in an exam context. However, GE must begin by reducing to zero all the elements below the diagonal entry of the first row, followed by all the elements below the diagonal entry of the second row and so on. GE is a relentlessly systematic method, and I expect the algorithm to be followed precisely for full marks. Anything other than this will attract a maximum of half marks, even if the answer is correct.

Row and column manipulations will speed up determinant evaluations, often by a considerable amount.

Regarding eigenvalues and eigenvectors, the setting of $\det(A-\lambda I) = 0$ will result in an equation for the eigenvalues, λ. It is then necessary to solve $(A - \lambda I)\underline{x} = \underline{0}$ to find the corresponding eigenvectors. For a 3×3 matrix, $A - \lambda I$ may have two rows which are either obviously identical or else they are obvious multiples of one another (although this isn't universal). Thus we only have two equations in three unknowns, and so one of the variables be set to an arbitrary value, and then we have two equations in two unknowns to solve.

Numerical Mathematics

You will need to remember (i) the formula for the Newton-Raphson method, and (ii) how to rearrange an equation to form at least one *ad hoc* iteration scheme to determine its roots.

Sometimes it may be necessary to use either the binomial expansion or a Taylor series to assist in the analysis of the convergence of these iteration schemes. You may quote these series from the formula book.

Least Squares

You will be expected to be able to derive a least squares formula. Fitting $y = mx + c$ is the classic case which people often memorise, but I am very likely to ask for something else such as $y = bx + cx^2$. So you will need to know the theory and how to reproduce it in all cases, even for $y = mx + c$.

I will not ask for RMS values of the residual; this is simply too much arithmetic for a maths paper.

I won't give anything more complicated than two unknown constants and therefore the matrix for the unknown coefficients will be a 2×2 at the most.

Fourier Series

The Fourier Series formula will be given on the exam paper in the precise form in which you'll need to use it.

You will be asked to sketch the function. At this point it is worth identifying whether the given function is even or odd. If even then all the B_n coefficients must be zero for symmtry reasons. Likewise all the A_n terms are zero when the function is odd. I will use this Fourier series as the forcing function for an ODE example.

Finally.....my primary aim with this unit has been to teach you methods and therefore my exam papers are designed to test your abilities to apply these same methods. So the memorisation of facts such as $\mathcal{L}[\cos \omega t] = s/(s^2 + \omega^2)$ is only of use for the purpose of checking your working. Old exam questions are a very good guide for what to expect.