Department of Mechanical Engineering, University of Bath

Mathematics 2 ME10305 Sheet 0

The following rather interesting pieces of mathematics were found in this year's Maths 1 examination scripts. The nature of the errors varies substantially from the trivial to the utterly appalling. Determine what the examinees did incorrectly in each case.

Q1. $\dfrac{d\ln|3t|}{dt} = \dfrac{3}{t}$

Answer: Given that $\ln|3t| = \ln 3|t| = \ln 3 + \ln|t|$, and noting that $\ln 3$ is a constant, the derivative of $\ln|3t|$ is $1/t$.

Q2. $\ln|3t| \Rightarrow \dfrac{3}{3t}$

Answer: Prior to the exams I was asked if I wished for students to simplify fractions. My answer was in the negative, but this is an example which could easily have been done. I didn't remove marks but I tutted a bit.

Q3. $\displaystyle\int \cos^2\theta \, d\theta = \sin^2\theta$

Answer: This is a standard category of error where two different operations yield different results depending the order in which they are applied. Here, it has been assumed that the integral of a square is the same as the square of the integral. No, that's wrong.

Q4. $\displaystyle\int \cos^2\theta \, d\theta = \tfrac{1}{3}\cos^3\theta$

Answer: This appear to mimic the fact that the integral of θ^2 is $\frac{1}{3}\theta^3$. Not good.

Q5. $\displaystyle\int \cos^2\theta \, d\theta = \tfrac{1}{3}\sin^3\theta$

Answer: This is a bit like the previous one except that cosine has been integrated to become a sine. Not at all good.

Q6. $\displaystyle\int \cos^2\theta \, d\theta = \tfrac{1}{2}\cos^2\theta^2$

Answer: So it's integrating "inside the cosine squared" — no such thing — but the $\frac{1}{2}$ is then taken outside of the cosine squared. Very strange. That was a bad day in the office.

Q7. $\displaystyle\int \cos^2\theta \, d\theta = \tfrac{1}{3}\cos^3\theta\sin\theta$

Answer: This is an integral version of the chain rule. Yes, think about that! Very very dodgy.

Q8. $x^2 = 3x \Rightarrow x = 3$

Answer: This type of error happened a lot when finding critical points. In this case the intermediate step was to cancel x on both sides. Often this is perfectly valid, but in the context of critical points it is tantamount to throwing away information, the information being that $x = 0$ is also a root. The next step should have been a rearrangment followed by a factorisation: $x(x-3) = 0$.

Q9. assymtote at $x = 0$

Answer: Spelling mistake. Should be asymptotic. I had very very few of these this year.

Q10. Three saddles found — no max or min. Must be another point.

Answer: There is no error here. Rather, this is an example of student who knows instinctively that it is impossible to have a surface where all three critical points are saddles. There must be at least one maximum or minimum somewhere. I was very pleased and impressed to see that on a script.

Q11. $z = 13e^{j(\theta+2\pi n)}$ with $\theta = 67.38°$

Answer: Here θ is in degrees but $2\pi n$ is in radians. Generally these cannot be mixed. More pertinently, all angles must be in radians when dealing with complex exponentials.

Q12. Sorry for drawing in pen, I forgot I had my pencil with me.

Answer: Again, there was nothing wrong with this, apart from using a comma instead of a semi-colon. I was astonished to have been apologised to while the student was in the middle of writing an exam script with all the stress and tension that is involved. I just wanted to say thanks for the courtesy, and also for not using a 2H.

Q13. $\displaystyle\int_0^2 r^3\,dr = \Big[3r^2\Big]_0^2$

Answer: Oops, wrong direction. You've differentiated. I had depressingly many of those.

Q14. $|-x| < 1 \Rightarrow x < -1$ so there's an infinite radius of convergence

Answer: The error is in not realising that $|-x|$ is the same as $|x|$. Then we would have concluded that $|x| < 1$, and hence the series has a unit radius of convergence.

Q15. It is a maxima at $t = 1$. The critical point is a minima.

Answer: You've used the plural form of the words. These should be *maximum* and *minimum*. It is a good job that this happened on an anonymous exam script rather than during your talk to your boss and the international partners when on industrial placement.

Q16. $\underline{\mathrm{r}} \cdot \underline{\mathrm{b}} - \underline{\mathrm{a}} = \cdots$

Answer: A case where the absence of a pair of brackets makes the mathematical expression to be incorrect. Here we have a scalar with a vector being subtracted from it. That is not possible. The correct form should have been $\underline{\mathrm{r}} \cdot (\underline{\mathrm{b}} - \underline{\mathrm{a}}) = \cdots$

Q17. $2(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}) = 0.0350 + j\,3.190 \times 10^{-4}$

Answer: It took me a while to work this one out. I reckon that the student didn't recall the facts that the cosine and sine of $\pi/6$ are $\sqrt{3}/2$ and $1/2$, otherwise this answer couldn't have happened. I also reckon that the student had a calculator which was set in degrees, not radians, and that they didn't know how to change that calculator setting. The appropriate conversion factor is $\pi/180$. But instead of multiplying the angles by that factor and then taking the cosine and sine, the cosine and sine were taken first (using $\pi/6$ degrees) and the answers multiplied afterwards by the factor.

Q18. $\displaystyle e^x \sum_{n=0}^{\infty} = \frac{x^n}{n!}$

Answer: Looks like gibberish, but the summation sign and the equals sign need to be swapped.

Q19. Using intergration by parts

Answer: Spelling. Integration.

Q20. $\displaystyle I = \int_0^1 x^2 \ln x\,dx. \quad \text{Let } x = e^{-y} \quad \Longrightarrow I = -\int_{e^{-1}}^1 ye^{-3y}\,dy$

Answer: Many did this. Strictly, when $x = 1$ we need $y = 0$ and when $x = 0$ we have $y = \infty$. So that second integral should be from $+\infty$ to 0. I reckon that the transforming of the limits of integration was sort of done the other way around. The following is complete rubbish, but it's the only explanation that I can think of. When $y = 0$ then $x = 1$ and when $y = 1$ then $x = e^{-1}$. Eeeek!

Q21. $\displaystyle\int_0^1 x^3\Big[1 + 9x^4\Big]^{1/2} dx = \int_0^1 x^3\Big[1 + 3x^2\Big]\,dx$

Answer: This error is the same as saying that $(a^2 + b^2)^{1/2} = a + b$.

Q22. It is a stationary maximum.

Answer: No, it's just a maximum.

Q23. $72\lambda = 36 \Rightarrow \lambda = \frac{72}{36} = 2$

Answer: I have committed this sin many times too. So easy to do it....

Q24. Let $x = e^{-y}$ in x^2. Hence $e^{(-y)^2}$ or e^{-y^2}

Answer: Somehow this seems a million miles from $e^x \times e^x = e^{x+x} = e^{2x}$, but one does need to be careful when manipulating exponents.

Q25. $z(x,y) = xy(y+x-3) \quad \Longrightarrow z(x) = y^2 + 2xy - 3y$ and $z(y) = x^2 + 2xy - 3x$

Answer: Someone (no, about four or five) invented this notation. These should take one of the standard forms, either z_x and z_y, or $\partial z/\partial x$ and $\partial z/\partial y$.

Q26. $\dfrac{x^2 - 2xy - y^2}{(x^2+y^2)^2} = \dfrac{(x-y)^2}{(x^2+y^2)^2}$

Answer: Another oops moment.

Q27. Let $t = x$

Answer: I did have a student ask me if he could do this in the exam because he found it easier to differentiate with respect to x than with respect to t. I did say that this was ok but the ultimate aim should be to be able to do these things independently of the variable names. Why did I go soft on this? It reminded me of an exercise that I had to do for my A-level music teacher, which was to harmonise a short melody in C$\sharp$ minor in the style of a Bach chorale. I couldn't think easily in that key, but I could do so in D minor. So I transposed the melody up a semitone, applied my all skills with a vengeance and, once satisfied, transposed it all back to the original key. Not ideal, and I certainly didn't tell the teacher, but I've never needed to harmonise in the style of Bach since I left school. It's an anecdote and an allegory.

Q28. $\displaystyle\int \frac{1}{x^2}\,dx = \ln|x^2|$

Answer: No, no, definitely not. No.

Q29. Find $r = 2e^{j\pi/6}$. Let $z = a + bj$ Hence $r = 2$. So $\sqrt{a^2+b^2} = 2$. Also $\arg(z) = \frac{1}{6}\pi$. So $\tan\frac{1}{6}\pi = b/a = 1/\sqrt{3} \quad \Longrightarrow \quad a = \sqrt{3}b \quad \ldots\ldots \quad a = \sqrt{3}$ and $b = 1$.

Answer: I get a dozen of these every year. It is correct, but it takes a page to do. One then has to solve a quadratic for either b^2 or a^2 and so on. On the other hand, $2(\cos\frac{1}{6}\pi + j\sin\frac{1}{6}\pi)$ can be evaluated in just one line.