Department of Mechanical Engineering, University of Bath

Mathematics ME10305

Problem Sheet 2 — ODE solutions

Questions 1 and 2 contain equations most of which are of exam standard. Questions 3 and 4 are longer, and while they use some ideas (e.g. l'Hôpital's rule) which won't be examined, questions of this type may arise and have arisen in past exams. It is best to be guided by past exam papers in this regard.

Q1. First find the general solution of the following homogeneous equations. Then find the solution which satisfies $y(0) = 1$ and $y'(0) = 0$ (additionally $y''(0) = 0$ for third and fourth order equations and $y'''(0) = 0$ for fourth order equations).

(a) $\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 4y = 0$; (b) $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} + 4y = 0$; (c) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 5y = 0$; (d) $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 29y = 0$;

(e) $\dfrac{d^3y}{dt^3} + 2\dfrac{d^2y}{dt^2} + \dfrac{dy}{dt} + 2y = 0$; (f) $\dfrac{d^3y}{dt^3} + \dfrac{d^2y}{dt^2} - 2y = 0$; (g) $\dfrac{d^3y}{dt^3} - 3\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} - y = 0$;

(h) $\dfrac{d^4y}{dt^4} + 4y = 0$; (i) $\dfrac{d^4y}{dt^4} + 5\dfrac{d^2y}{dt^2} + 4y = 0$; (j) $\dfrac{d^4y}{dt^4} + 2\dfrac{d^2y}{dt^2} + y = 0$.

Q2. Find the general solution of the following inhomogeneous equations.

(a) $\dfrac{d^2y}{dt^2} + 9y = f(t)$ where $f(t)$ takes the following forms: (i) e^{at}, (ii) t^3, (iii) $\cos at$, (iv) $\cos 3t$.

(b) $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 2y = f(t)$ where $f(t)$ takes the following forms: (i) e^{at}, (ii) t^2, (iii) $\cos at$.

(c) $\dfrac{d^2y}{dt^2} - 7\dfrac{dy}{dt} + 12y = f(t)$ where $f(t)$ takes the following forms: (i) e^{2t}, (ii) e^{3t}, (iii) t^2 (iv) $\cos at$.

(d) $\dfrac{d^3y}{dt^3} + 3\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} + y = t^3e^{-t}$. (Do this first using the standard way, and then by using the substitution, $y(t) = z(t)e^{-t}$.)

Q3. Solve the equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = e^{2t}, \qquad y(0) = 0, \quad \left.\frac{dy}{dt}\right|_{t=0} = 0,$$

using standard CF/PI methods.

Now we will attempt to solve the same equation using a slightly different method. Let us first find the solution of

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = e^{at}, \qquad y(0) = 0, \quad \left.\frac{dy}{dt}\right|_{t=0} = 0,$$

where $a \neq 2$. Now let $a \to 2$ in the answer, and use L'Hôpital's rule to recover the solution when $a = 2$.

Q4. In this question the equation,

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = te^{-2t},$$

will be solved in two different ways.

(a) Use the Complementary Function/Particular Integral approach.

(b) Use the substitution $y = z(t)e^{-2t}$ to simplify the equation. You should then be able to integrate the resulting equation once with respect to t. The final first order equation for z may then be solved using the CF/PI approach.

D.A.S.R. 29/01/2016