Department of Mechanical Engineering, University of Bath

ME10305 Mathematics 2

Laplace Transforms Sheet 0

This problem sheet is an experiment to see if you can solve an ordinary differential equation using Laplace Transforms before I even begin lecturing on the topic. It's your choice if you wish to rise to the challenge. I'll try to walk you through it gently.

Here's the definition. If we have a function of time, $f(t)$, then its Laplace Transform is defined to be $F(s)$ where

$$F(s) = \mathcal{L}\big[f(t)\big] = \int_0^\infty f(t)e^{-st}\,dt,$$

where $\mathcal{L}\big[\quad\big]$ is just a mathematical shorthand for saying the words, 'Laplace Transform of...'. Don't worry about what all of this means — it's really weird — just go with the flow for now.

Q1. Use the definition of the Laplace Transform to find $\mathcal{L}\big[e^{-at}\big]$. This should be a nice quick integral, and your answer should come out to be $1/(s+a)$.

Q2. As the aim is solve a differential equation we had better find an expression for $\mathcal{L}\big[dy/dt\big]$.

So let $Y(s) = \mathcal{L}\big[y(t)\big]$, and apply the Laplace Transform formula to dy/dt.

You will need one integration by parts to do this, and the final answer will involve both $Y(s)$ and the value of y at $t = 0$, i.e. $y(0)$. In the context of ODEs the value of $y(0)$ represents the initial condition.

Q3. Believe it or not, we are now in a position to solve an ODE. So use both of the above results to find the Laplace Transform of the ordinary differential equation,

$$\frac{dy}{dt} + 2y = e^{-t}, \qquad \text{subject to } y(0) = 2.$$

Once the equation (with the boundary condition) has been transformed, first rearrange it to find $Y(s)$ explicitly, then use partial fractions to simplify that expression, and then finally use the result in Q1 to find the function, $y(t)$, which corresponds to your $Y(s)$.

Q4. Perhaps you should solve the equation using the CF/PI method (i) to ensure that you can(!) and (ii) to check that the Laplace Transform solution is correct.

D.A.S.R. 08/02/2016