Department of Mechanical Engineering, University of Bath

Mathematics 2 ME10305

Problem sheet — Least Squares Fitting of data

1. The Maths 2 webpage has links to the last five years of exam questions. Bizarrely, I think that the Least Squares questions on those papers are the best place to start this problem sheet! So do have a go at these before any of the other questions here, for they will, at least, inform you of precisely what you should expect in terms of the length of a question and how much calculator work I will expect.

2. In this question we are going to play a different game in the sense that the data to be fitted has a different type of randomness. I would like to see how much accuracy one can gain from a set of data which has been rounded quite severely. We'll use the conversion from miles to kilometres for this purpose. The following is a Table of data which is subject to different degrees of rounding off, namely to zero, 1, 2 and 3 decimal places; x denotes miles and yn denotes kilometres with n decimal places.

For each of these sets of data, fit a straight line through the origin to determine the least squares version of the conversion factor between the units. How accurate are they? You may compare with the exact value, 1.609344.

x :	1	2	3	4	5	6	7	8	9	10
$y0$:	2	3	5	6	8	10	11	13	14	16
$y1$:	1.6	3.2	4.8	6.4	8.0	9.7	11.3	12.9	14.5	16.1
$y2$:	1.61	3.22	4.83	6.44	8.05	9.66	11.27	12.87	14.48	16.09
$y3$:	1.609	3.219	4.828	6.437	8.047	9.656	11.265	12.875	14.484	16.093

If this has piqued your interest, then try the same with the conversion between ounces and grams, for which 1 ounce is equal to 28.349523 grams. Use 16 sets of data from 1oz to 16oz, and use the same number of decimal places as in the above miles/kilometres example. The raw data may be found at

`http://staff.bath.ac.uk/ensdasr/ME10305.bho/ounces-to-grams.txt`

although there is also a link to it at the unit webpage. Given that there are 16 data points, you might be able to coerce Excel into doing your calculations for you.

3. Suppose that you were given a set of experimental data where it is suspected that the data should satisfy an equation of the form

$$y = a + b/x.$$

How could the data be manipulated in order to use standard Least Squares theory? [Note: I can think of at least two different ways of doing this.]

What about the equation, $$y = \frac{a}{x+b}?$$

4. In many experimental situations the observable, y, is a power law function of the parameter, x. In other words it takes the form,

$$y = ax^b,$$

where a and b need to be found. [For example, the rate of heat transfer from a hot vertical surface is proportional to the $\frac{1}{4}$ power of the temperature difference between the heated surface and the ambient conditions.] How would you convert this power-law relationship into a straight line relationship?

5. Experimental measurements have been taken of z, which is a function of both x and y. It is suspected that z is a linear function of x and y, and therefore it represents a plane in 3D space. Use least squares theory to determine the three unknown coefficients in the following equation for the plane,

$$z = ax + by + c.$$

6. An obsessive cyclist has a comprehensive set of data for his/her ride-times over the same route for a period of several years. Naturally the cyclist's journey times are slower when the weather is colder, and faster when it is warmer. The cyclist wishes to determine (i) what the long term general trend is in terms of speed, (ii) what seasonal effect should be expected given the time of year. To this end, the cyclist proposes a least squares fit of the form,

$$T = a + bt + c\cos(2\pi t) + d\sin(2\pi t)$$

where T is the ride-time and t is time measured in years. Develop the least squares theory which will allow the cyclist to achieve his/her twin objectives.

7. An experiment has two measurables, $y(x)$ and $z(x)$, as the control parameter, x, is varied. Both y and z should be linear with different slopes, but should have the same intercept on the vertical axis. That is, we wish to fit the following to the data, where there are three constants to find:

$$y = ax + c, \qquad z = bx + c.$$

How is this done? [This is a simplified version of a problem a couple of third year students brought to me where they had to fit a straight line to five measureables all of which had the same intercept. So this question isn't a product of my wild imaginings! What was the answer for their problem?]

D.A.S.R. 25/03/2021