

## RADIOMETRY

RADIOMETRY - MEASUREMENT OF INCOHERENT ELECTROMAGNETIC ENERGY

- ⊗ ALL MATTER WHETHER GAS, VAPOUR, SOLID OR PLASMA EMITS E.M. ENERGY.
- ⊗ THERMAL EMISSION IS THE DOMINANT PROCESS.
- ⊗ A RADIOMETER IS A VERY SENSITIVE, LOW-NOISE RECEIVER THAT CAN DETECT THIS INCOHERENT E.M. ENERGY.

APPLICATIONS OF RADIOMETRY INCLUDE:

- ⊗ MEASUREMENT OF THE ATMOSPHERE.
- ⊗ SECURITY - OBJECTS HIDDEN BENEATH/ UNDER CLOTHES.
- ⊗ MEDICAL - CLASSIFICATION OF TISSUE TYPES.

THE FUNDAMENTAL QUANTITY IN RADIOMETRY IS THE BRIGHTNESS,  $B$

BRIGHTNESS HAS UNITS OF  $\text{W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}$ .

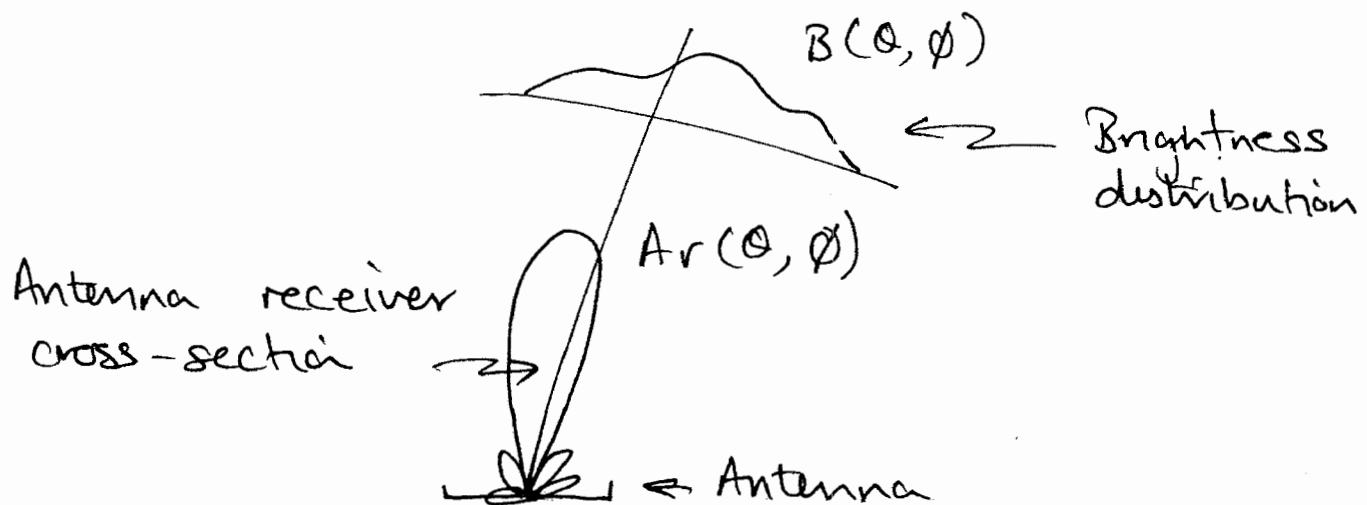
(KNOWN AS RADIANCE IN OPTICAL RADIOMETRY)

- $dP$  - POWER  
 $da$  - AREA  
 $B$  - BRIGHTNESS  
 $d\Omega$  - SOLID ANGLE  
 $df$  - FREQUENCY

$$dP = B \cos\theta da d\Omega df.$$

$B$  - DEPENDS ON DIRECTION -  $B(\theta, \phi)$   
BRIGHTNESS DISTRIBUTION.

$B$  - DEPENDS ON FREQUENCY -  $B(f)$   
BRIGHTNESS SPECTRUM.



$$P_r = \frac{1}{2} \int_{4\pi} B(\theta, \phi) Ar(\theta, \phi) d\Omega$$

$\frac{1}{2}$  - ANTENNA IS POLARIZED - INCOHERENT  
RADIATION IS UNPOLARIZED

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IF WE NORMALIZE THE RECEIVING  
CROSS-SECTION  $A_r$  TO THE MAXIMUM  
VALUE WE HAVE;

$$P_r = \frac{1}{2} A_{rm} S,$$

$$S = \int_{4\pi} B(\theta, \phi) P_n(\theta, \phi) d\Omega$$

$$A_{rm}(\theta, \phi) = A_{rm} P_n(\theta, \phi)$$

$A_{rm}$  CAN BE WRITTEN IN TERMS OF  
ANTENNA GAIN ( $G_m$ )

$$A_{rm} = \frac{\lambda^2}{4\pi} G_m.$$

$$S_s = \int_{4\pi} B(\theta, \phi) d\Omega$$

IS THE SOURCE FLUX DENSITY  $\text{W m}^{-2} \text{Hz}^{-1}$ ,

RADIO ASTRONOMERS CALL THIS BY ANOTHER  
UNIT THE JANSKY.  $1 \text{jan} = 1 \text{W Hz}^{-1} \text{m}^{-2}$   
NAMED AFTER KARL G. JANSKY.

MOST RADIO SOURCES IN RADIO ASTRONOMY  
ARE OF ORDER  $10^{-26}$  jan.

## BLACK BODY RADIATION

- ④ KIRCHHOFF'S LAW : A GOOD ABSORBER OF EM ENERGY IS A GOOD EMITTER.
- ⑤ IN GENERAL RADIATION INCIDENT ON A MATERIAL WILL BE REFLECTED AND ABSORBED.
- ⑥ A BLACK BODY MATERIAL - ABSORBS RADIATION AT ALL FREQUENCIES REFLECTING NONE. - IDEALIZED
- ⑦ A PERFECT ABSORBER IS A PERFECT EMITTER.
- ⑧ ACCORDING TO PLANCK'S RADIATION LAW A BLACK BODY RADIATES UNIFORMLY IN ALL DIRECTIONS WITH A SPECTRAL BRIGHTNESS  $B_f$  GIVEN BY:

$$B_f = \frac{2hf^3}{c^2} \frac{1}{\exp(hf/kT) - 1}$$

- $h$  - PLANCK'S CONSTANT -  $6.63 \times 10^{-34}$  (Js)
- $f$  - FREQUENCY (Hz)
- $k$  - BOLTZMANN'S CONSTANT -  $1.38 \times 10^{-23}$  ( $\text{J K}^{-1}$ )
- $T$  - TEMPERATURE (K)
- $c$  - VELOCITY OF LIGHT -  $3 \times 10^8$  ( $\text{m s}^{-1}$ )

FOR MICROWAVE & MILLIMETRE WAVES,

$$hf \ll kT.$$

$$\exp(hf/kT) \approx 1 + hf/kT$$

THIS APPROXIMATION IS CALLED THE  
RAM LEIGHT - JEANS LAW;

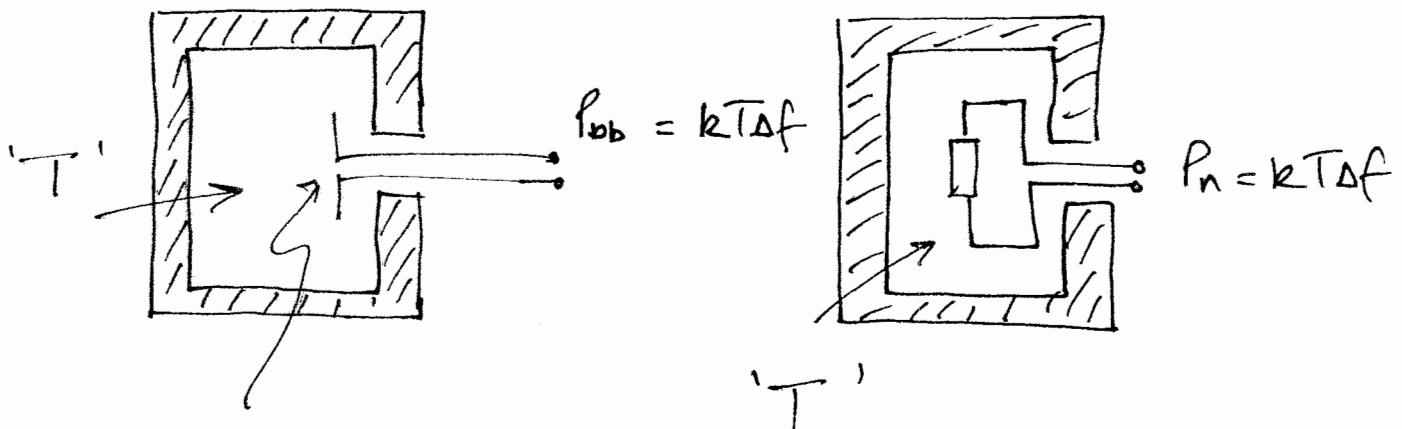
$$B_f = \frac{2k}{\lambda^2} T.$$

STEFAN - BOLTZMANN LAW - TOTAL  
BRIGHTNESS AT TEMPERATURE T

$$B = \int_0^\infty B_f df = \frac{\sigma T^4}{\pi}$$

$\sigma$  - STEFAN BOLTZMANN CONSTANT  
 $5.673 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \text{ sr}^{-1}$ .

POWER - TEMPERATURE CORRESPONDENCE



ANTENNA A

$$P_{bb} = kTdf \frac{Ar}{\lambda^2} \iint f_n(\theta, \phi) d\Omega$$

$$R_A = \iint_{4\pi} f_n(\theta, \phi) d\Omega = \frac{\lambda^2}{A_r}$$

HENCE  $f_{bb} = kT_B \Delta f = f_n$ .

### NON BLACKBODY RADIATION

NON BLACKBODIES CALLED "GREY BODIES"

BRIGHTNESS OF BLACKBODY:

$$B_{bb} = B_f \Delta f = \frac{2kT}{\lambda^2} \Delta f.$$

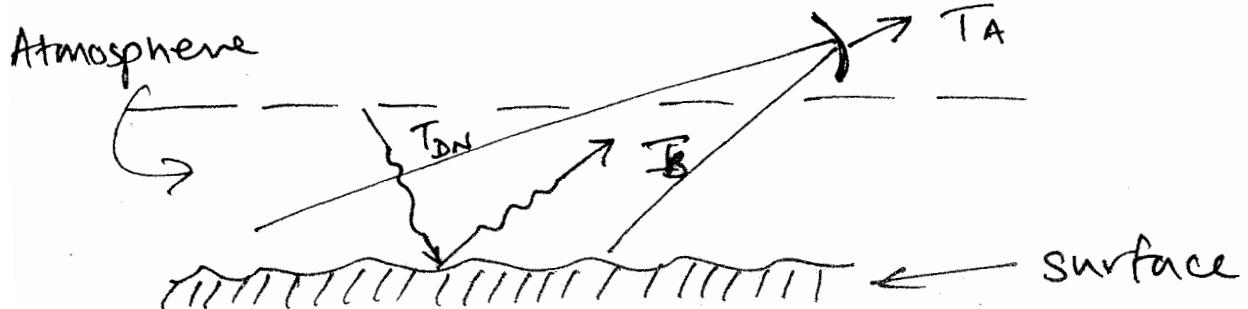
FOR A NARROW BANDWIDTH,  $\Delta f$ .

$$B(\theta, \phi) = \frac{2k}{\lambda^2} T_B(\theta, \phi) \Delta f.$$

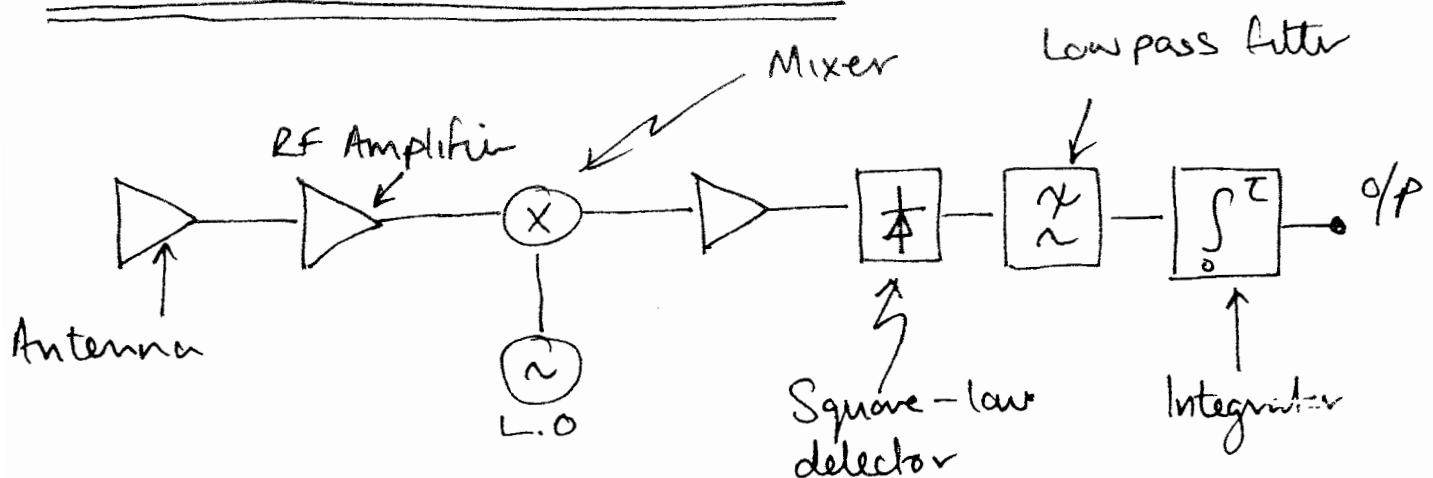
$$e(\theta, \phi) = \frac{B(\theta, \phi)}{B_{bb}} = \frac{T_B(\theta, \phi)}{T}.$$

$e(\theta, \phi)$  - EMISSIVITY  $0 \leq e(\theta, \phi) \leq 1$ .

TO SENSE  $T_B$  WE USE AN ANTENNA AND MEASURE ITS TEMPERATURE,  $T_A$   
IDEALLY  $T_A = T_B$



## TOTAL POWER RADIOMETER



POWER MEASURED CONSISTS OF,  $T_{sys}$ , SYSTEM TEMPERATURE (BACKGROUND) AND SIGNAL TEMPERATURE  $\Delta T$ .

SQUARE-LAW DETECTOR - OUTPUT VOLTAGE PROPORTIONAL TO OUTPUT NOISE POWER

OUTPUT FROM SYSTEM NOISE =  $(k T_{sys} \text{BW})^2$   
 BW - BANDWIDTH OF RECEIVER.

AVERAGED BY INTEGRATION =  $BW C$ .  
 SIGNAL OUTPUT =  $(k \Delta T \text{BW})$

$$V_{sys} \approx \frac{(k T_{sys} \text{BW})^2}{BW C} - \text{OUTPUT VOLTAGE SIGNAL.}$$

EFFECTIVE REDUCTION IN NOISE TEMPERATURE FROM  $T_{sys}$  TO  $T_{sys} / (BW C)^{1/2}$

$$\text{SIGNAL OUTPUT } V_s \approx (k \Delta T \text{BW})^2$$

$$V_s = V_{sys} \Rightarrow \Delta T = \Delta T_{min}$$

$$\Delta T_{min} = \frac{T_{sys}}{\sqrt{BW C}}$$

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IN GENERAL;

$$\overline{V_{out}} = g_{LF} C_d G k T_{sys} \Delta f$$

$\Delta f$  - BANDWIDTH

$T_{sys}$  - TEMPERATURE

$k$  - BOLTZMANN'S CONSTANT

$G$  - SYSTEM GAIN

$C_d$  - DETECTOR SENSITIVITY ( $V W^{-1}$ )

$g_{LF}$  - LOW-PASS FILTER GAIN.

THE PROBLEM:

$$\overline{V_{out}} \propto G T_{sys}.$$

$G_s \rightarrow \Delta G_s + G_s$  AS GAIN VARIES  
LEADS TO INCREASE IN  $T_{sys}$  BY

$$\Delta T_g = \Delta T_{sys} = T_{sys} \left( \frac{\Delta G_s}{G_s} \right)$$

SINCE NOISE FLUCTUATIONS ( $\Delta T_{min}$ ) AND GAIN FLUCTUATIONS ARE UNCORRELATED, TOTAL UNCERTAINTY (RMS.) GIVEN BY;

$$\begin{aligned} \Delta T &= \left[ (\Delta T_{min})^2 + (\Delta T_g)^2 \right]^{1/2} \\ &= T_{sys} \left[ \frac{1}{\Delta f} + \left( \frac{\Delta G_s}{G_s} \right)^2 \right]^{1/2}. \end{aligned}$$

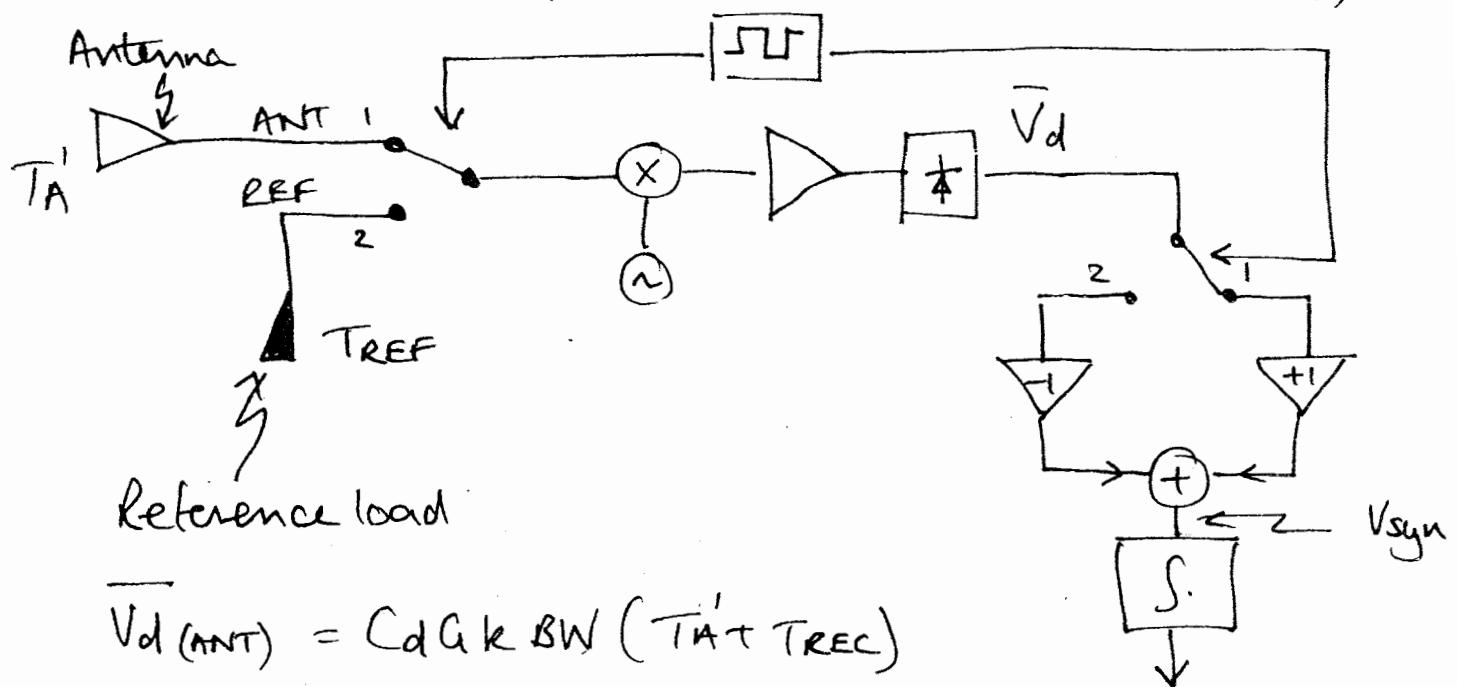
$$T_{sys} = 600K, \Delta f = 100MHz, \Delta f = 10ms, \Delta G_s/G_s = 0.01$$
$$T_A = 300K \Rightarrow \Delta T_{min} = 0.9K, \Delta T_g = 9K, \Delta T = 9.05K$$

GAIN VARIATIONS CAN DOMINATE SENSITIVITY  
 → REDUCE GAIN FLUCTUATIONS!

$\Delta G_s/G_s < 10^{-5}$  CAN BE ACHIEVED,  
 BUT CAN BE COSTLY PARTICULARLY AT  
 MILLIMETER WAVELENGTHS.

### DICKE RADIOMETER

R.H DICKE (1946) SOLVED THIS PROBLEM;



$$\bar{V}_d(\text{ANT}) = C_d G k \text{BW} (\bar{T}_n' + T_{\text{REC}})$$

$$\bar{V}_d(\text{REF}) = C_d G k \text{BW} (T_{\text{REF}} + T_{\text{REC}})$$

$$\bar{V}_{\text{syn}} = \frac{1}{2} (\bar{V}_d(\text{ANT}) - \bar{V}_d(\text{REF}))$$

$$= \frac{1}{2} C_d G k \text{BW} (\bar{T}_n' - T_{\text{REF}})$$

IF  $\bar{T}_n' = T_{\text{REF}}$  GAIN FLUCTUATIONS ARE NIL.

$$\Delta T_{\text{MIN}} = \left[ \frac{2(\bar{T}_n' + T_{\text{REC}})^2 + 2(T_{\text{REF}} + T_{\text{REC}})^2}{\text{BW} \tau} + \frac{\Delta G_s (T_n - T_{\text{REF}})}{G_s} \right]^{1/2}$$

FOR A BALANCED DICKE RADIOMETER

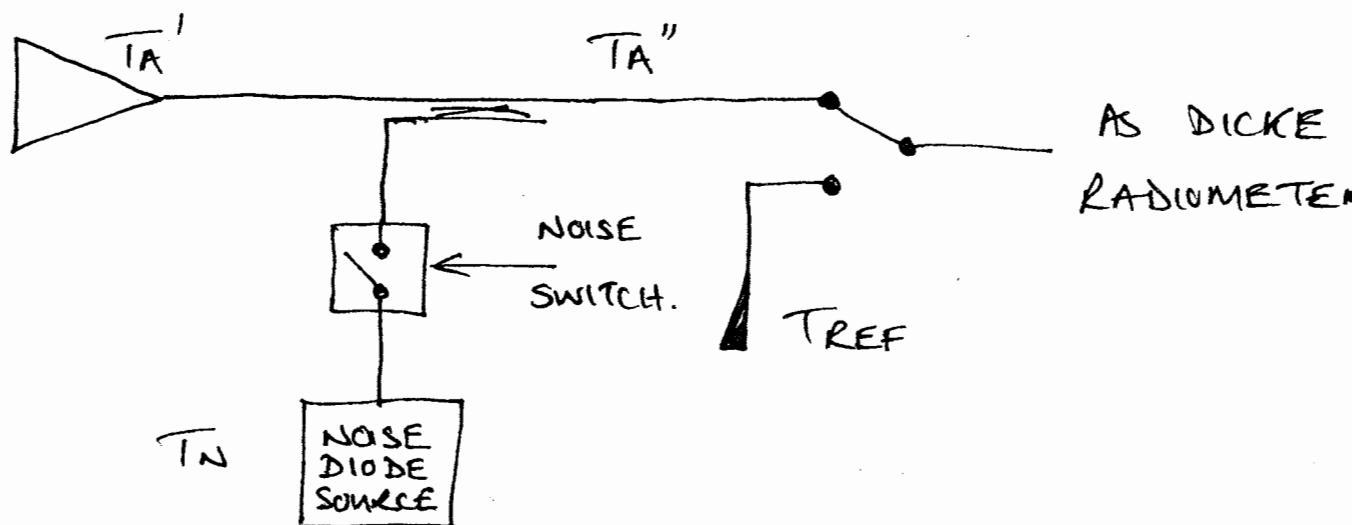
$\overline{T_A}' = \overline{T_{REF}}$ , SENSITIVITY CAN BE WRITTEN AS;

$$\Delta T_{MIN} = \frac{2(\overline{T_A} + \overline{T_{REC}})}{\sqrt{BW\tau}}$$

$$T_{sys} = \overline{T_A} + \overline{T_{REC}}$$

$$\Delta \overline{T}_{MIN} \underset{(DICKE)}{=} 2 \Delta \overline{T}_{MIN} \underset{(IDEAL)}{=}$$

### PULSED NOISE INJECTION BALANCING



AT BALANCE  $\overline{T_A}'' = \overline{T_{REF}} \Rightarrow \overline{V_{syn}} = 0$ .

$$\left( \overline{T_A}' + \alpha T_N \right) = \overline{T_A}'' = \overline{T_{REF}}$$

ADJUST  $\alpha$  BY PULSING NOISE SWITCH.