# WAVE PROPAGATION AND SCATTERING IN RAIN

AS WE HAVE SEEN EARLIER THE TROPOSPHERE CAN BE SPLIT INTO THREE BASIC REGIONS

- RAIN REGION $\simeq 0 - 4$ km
- MELTING LAYER $\simeq 2$-4km – FEW HUNDRED m THICK.
- ICE REGION $> 4$ km – TO THE TROPOPAUSE

IN CONSIDERING TROPOSPHERIC POINT – TO POINT LINKS OUR MAIN CONCERN IS RAIN

FOR SATELLITE EARTH - SPACE, SPACE - EARTH LINKS WE ARE INTERESTED IN ALL THREE LAYERS.

IN ORDER TO ESTIMATE THE ATTENUATION DUE TO RAIN WE NEED TO KNOW;

- SIZE AND SHAPE OF RAINDROPS (p2)
- TEMPERATURE (REFRACTIVE INDEX)
- SIZE DISTRIBUTION OF RAINDROPS
- WAVELENGTH.

AND . . .

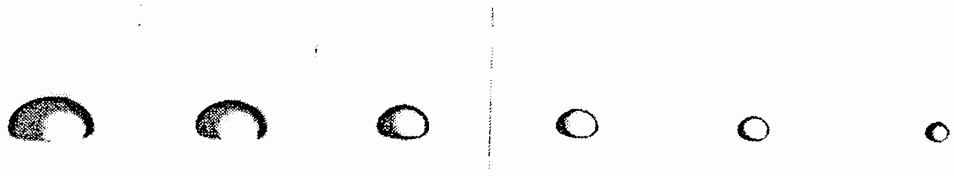


Fig. 4. Photographs of water drops of various sizes falling at their terminal velocities in air. From left to right, the equivolumetric sphere radius: 4.00, 3.675, 2.90, 2.65, 1.725, and 1.35 mm. (After Pruppacher and Beard [24]. Reprinted with permission from the Royal Meteorological Society, UK.)

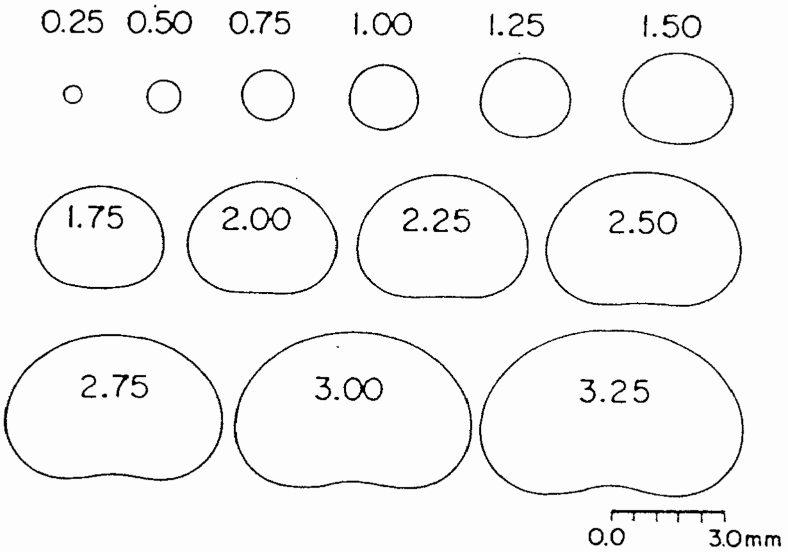


Fig. 6. Calculated drop shapes of 13 water drops. The numbers indicat equivolumetric sphere radius.

WE NEED TO KNOW THE SCATTERING PROPERTIES OF RAINDROPS.

WHEN AN ELECTROMAGNETIC WAVE IS INCIDENT ON A RAINDROP SOME OF THE ENERGY IS ABSORBED, AND SOME IS SCATTERED (OR RE-RADIATED)

TO MAKE THE CALCULATIONS OF THE SCATTERING TRACTABLE WE MAKE SOME ASSUMPTIONS ABOUT THE SHAPE AND SIZE OF THE RAINDROPS. FROM PAGE 2 WE CAN SEE THAT WE COULD MODEL ACTUAL RAINDROPS BY OBLATE SPHEROIDS WITH MAJOR AXES $a$, AND MINOR AXES $b$, RELATED TO THE EQUIVALENT VOLUME RADIUS $\bar{a}$.

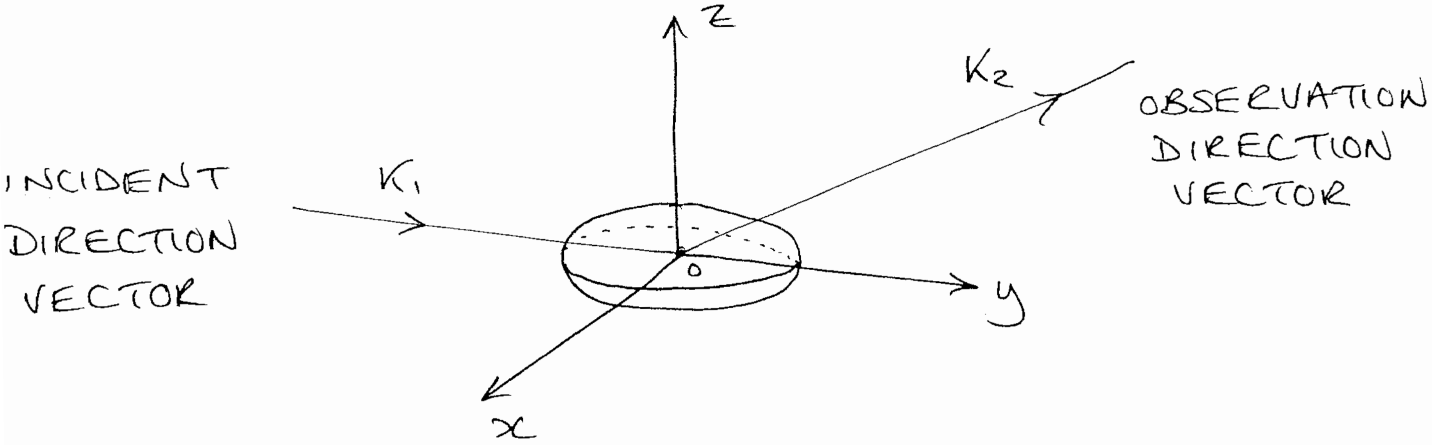
A NUMBER OF RELATIONSHIPS HAVE BEEN PROPOSED;

$$\frac{b}{a} = 1.03 - 1.24\bar{a} \quad \text{- PRUPPACHER \& BEARD (1970)}$$

$$\frac{b}{a} = 1.0 - \bar{a} \quad \text{- MORRISON \& CROSS (1974)}$$

PLUS OTHERS...

SUPPOSE WE LOOK AT A SINGLE RAINDROP;



THE TOTAL ELECTRIC FIELD AT A LARGE DISTANCE r (IN THE FAR FIELD) CAN BE WRITTEN AS

$$\underset{\substack{\nearrow \\ \text{TOTAL E-FIELD}}}{E(r)} = \underset{\substack{\uparrow \\ \text{INCIDENT}}}{E_0 \exp(-jk_0 r)} + \underset{\substack{\uparrow \\ \text{SCATTERED}}}{\frac{f(K_1,K_2)}{r}} \exp(-jk_0 r)$$

$f(K_1,K_2)$ – IS THE <u>SCATTERING AMPLITUDE</u>

FOR PROPAGATION WE ARE INTERESTED MAINLY IN <u>FORWARD</u> SCATTERING, THAT IS

$K_1 = K_2 \qquad f(K_1,K_2)$

FOR RADAR APPLICATIONS WE ARE INTERESTED MAINLY IN <u>BACKWARD</u> SCATTERING, THAT IS

$K_1 = -K_2 \qquad f(K_1,-K_1)$

SINCE WE USUALLY WANT TO WORK WITH POWERS WE ARE INTERESTED IS CROSS-SECTIONS (MORE ON THIS WHEN WE LOOK AT RADAR SYSTEMS)

THE COMBINATION OF THE ABSORPTION AND SCATTERING PROCESSES IS CALLED EXTINCTION

EXTINCTION = SCATTER + ABSORPTION

## SCATTERING CROSS-SECTION

$$\sigma_S = \frac{P_S}{\overline{S_I}}$$

TOTAL POWER SCATTERED BY THE RAINDROP

AVERAGE POWER DENSITY OF THE INCIDENT WAVE

## ABSORPTION CROSS-SECTION

$$\sigma_A = \frac{P_A}{\overline{S_I}}$$

POWER ABSORBED BY PARTICLE

AVERAGE POWER DENSITY OF THE INCIDENT WAVE

## EXTINCTION (ATTENUATION) CROSS-SECTION

$$\sigma_E = \sigma_S + \sigma_A$$

THE EXTINCTION CROSS-SECTION CAN BE EXPRESSED IN TERMS OF THE FORWARD SCATTERING AMPLITUDE;

$$\sigma_E = -\left(\frac{4\pi}{k_0}\right) \mathrm{Im}\left[f(K_1, K_2)\right]$$

FOR DROPS THAT ARE SMALL COMPARED TO THE WAVELENGTH WE CAN WRITE,

$$\sigma_E = -4\pi k_0 \, \mathrm{Im}\left[(\epsilon_r \epsilon_0 - 1)/(\epsilon_r \epsilon_0 + 2)\right] a^3$$

THIS IS THE <u>RAYLEIGH</u> APPROXIMATION VALID FOR $k_0 a \ll 1$

FOR RAIN $a_{MAX} \simeq 4mm$ $k_0 \simeq 1.26\,cm^{-1} \Rightarrow f \simeq 5GH$

FOR HIGHER FREQUENCIES;

- * MIE - (SPHERES ONLY)
- * T- MATRIX WATERMAN (1965)
- * FREDHOLM- INTEGRAL METHOD HOLT (1978)
- * FINITE - ELEMENT. MORGAN (1980)

THE VARIATION IN WAVE INTENSITY PASSING THROUGH A WAVE MEDIUM CAN BE WRITTEN AS

$$\frac{dI}{dr} = -I \sum \sigma_E ,$$

WHERE $\sum \sigma_E$ IS THE SUM OF THE CROSS-SECTIONS OF ALL THE RAINDROPS IN A UNIT VOLUME OF SPACE.

THE SOLUTION OF THIS EQUATION IS;

$$I = I_0 \exp[(-\Sigma \sigma_E) r]$$

$I_0$ IS THE INTENSITY AT $r=0$

WE CAN WRITE THE SUM OF THE CROSS-SECTIONS AS;

$$\sum \sigma_E = \int_0^\infty \sigma_E(a) n(a) \, da$$

$n(a) \, da$ IS THE <u>RAINDROP SIZE DISTRIBUTION</u>

COMBINING ALL OF THIS, WE CAN WRITE THE ATTENUATION IN dB km$^{-1}$ AS

$$A = 10^3 \times \log_{10} e \times \int_0^\infty \sigma_E(a) n(a) da.$$

$$\simeq 4343 \int_0^\infty \sigma_E(a) n(a) da \quad \text{dB km}^{-1}$$

# RAINDROP SIZE DISTRIBUTION

THIS IS THE BIGGEST UNKNOWN BY FAR!

THE DROPSIZE DISTRIBUTION DENOTED $n(a)da$ IS THE AVERAGE NUMBER OF RAINDROPS PER UNIT VOLUME OF RADIUS $a$ to $a+da$.

FIRST MEASUREMENTS OF DSD: LAWS & PARSONS 1943

EXPONENTIAL RELATIONSHIP PROPOSED BY MARSHALL & PALMER, 1948.

$$n(a)\,da = N_o \exp(-\Lambda a)\,da$$

$$N_o = 1.6\times10^4 \ m^{-3}\,mm^{-1}, \quad \Lambda = 8.2R^{-0.21} \ mm^{-1}$$

JOSS, THAMS & WALDVOGEL 1968;

$$N_o = 6\times10^4 \ m^{-3}\,mm^{-1}, \quad \Lambda = 11.4R^{-0.21} \ mm^{-1} \quad \text{(DRIZZLE)}$$

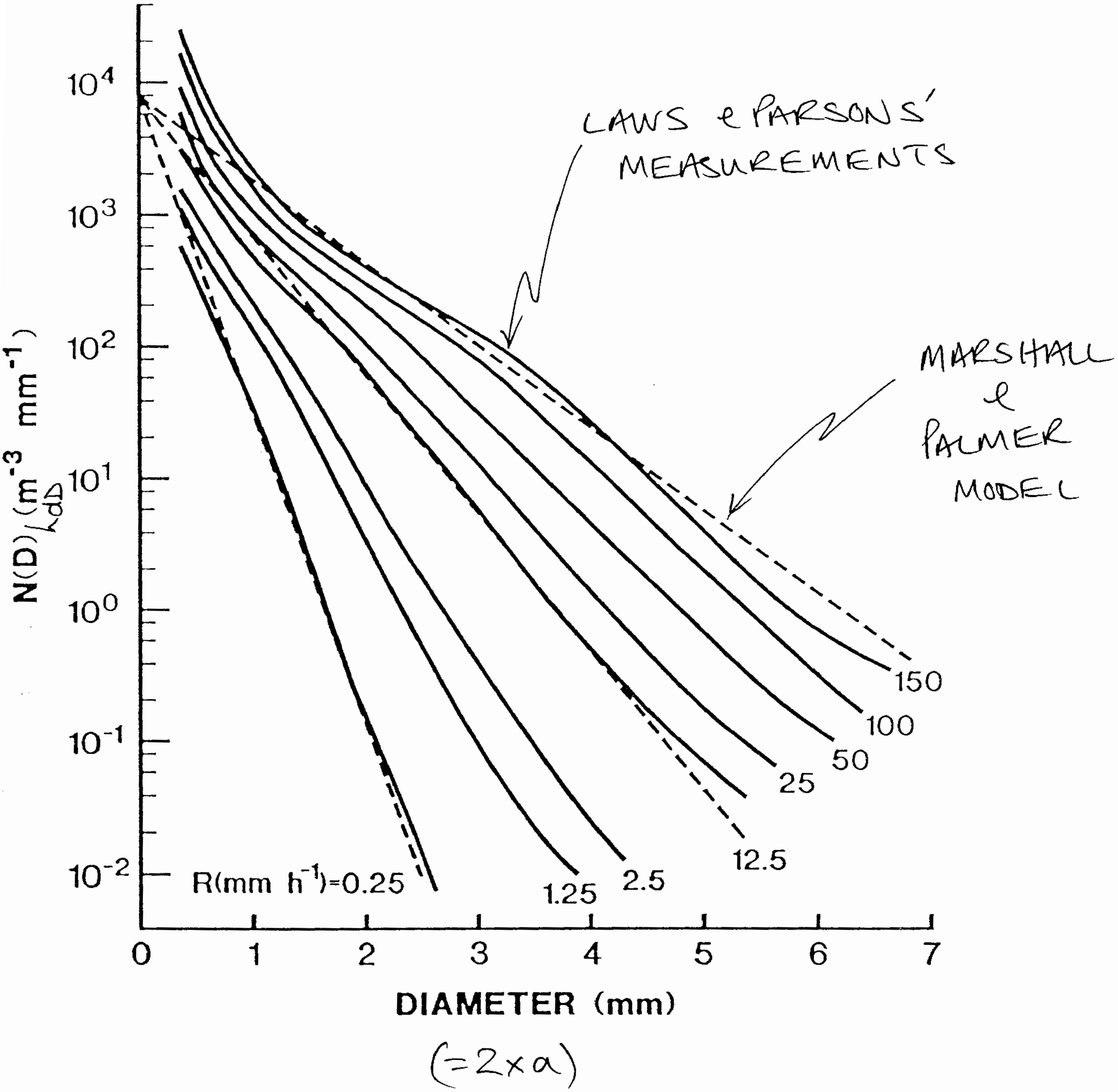
$$N_o = 1.4\times10^4 \ m^{-3}\,mm^{-1}, \quad \Lambda = 8.2R^{-0.21} \ mm^{-1} \quad \text{(WIDESPREAD)}$$

$$N_o = 2.8\times10^3 \ m^{-3}\,mm^{-1}, \quad \Lambda = 6.0R^{-0.21} \quad \text{(T/STORM)}$$

ULBRICH 1983;

$$n(a)\,da = N_o D^{\mu} \exp\left(-(3.67+\mu)\overset{=2a}{D}/D_o\right) \qquad N_o = 6\times10^4 \exp(3.2\mu)$$

$$-1 \le \mu \le 5 \qquad\qquad (m^{-3}\,mm^{-1})$$


LAWS & PARSONS' MEASUREMENTS
MARSHALL & PALMER MODEL
N(D) (m-3 mm-1)
/dD
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
R(mm h-1)=0.25
1.25
2.5
12.5
25
50
100
150
0
1
2
3
4
5
6
7
DIAMETER (mm)
(=2×a)

$$n(a)da = N(D)dD$$

## RAINFALL RATE

WE CAN DEFINE RAINFALL RATE $R\,(\mathrm{mm\,h^{-1}})$ IN TERMS OF $n(a)da$ ($= N(D)\,dD$) AS

$$R = \frac{\pi}{6}\int_0^\infty D^3 N(D)\, W_t(D)\, dD$$

$W_t(D)$ IS THE TERMINAL FALL-VELOCITY OF THE RAINDROP; FROM MEASUREMENTS IT WAS FOUND THAT $W_t(D)$ CAN BE APPROXIMATED BY;

$$W_t(D) = 9.65 - 10.3\exp(-600D) \quad \mathrm{ms^{-1}}$$

$$6\times10^{-4} \leq D \leq 5.8\times10^{-3}\,\mathrm{m}$$

HENCE GIVEN $N(D)dD$ (OR $n(a)da$) WE CAN ESTIMATE THE RAINFALL RATE.

## RAINRATE - ATTENUATION CURVES

GIVEN: - WAVELENGTH
- DROP SHAPE MODEL
- RAINDROP SIZE DISTRIBUTION
- REFRACTIVE INDEX OF WATER (FUNCTION OF TEMPERATURE)

WE CAN COMPUTE $A$ $\mathrm{dB\,km^{-1}}$ AND $R$ $\mathrm{mm\,h^{-1}}$

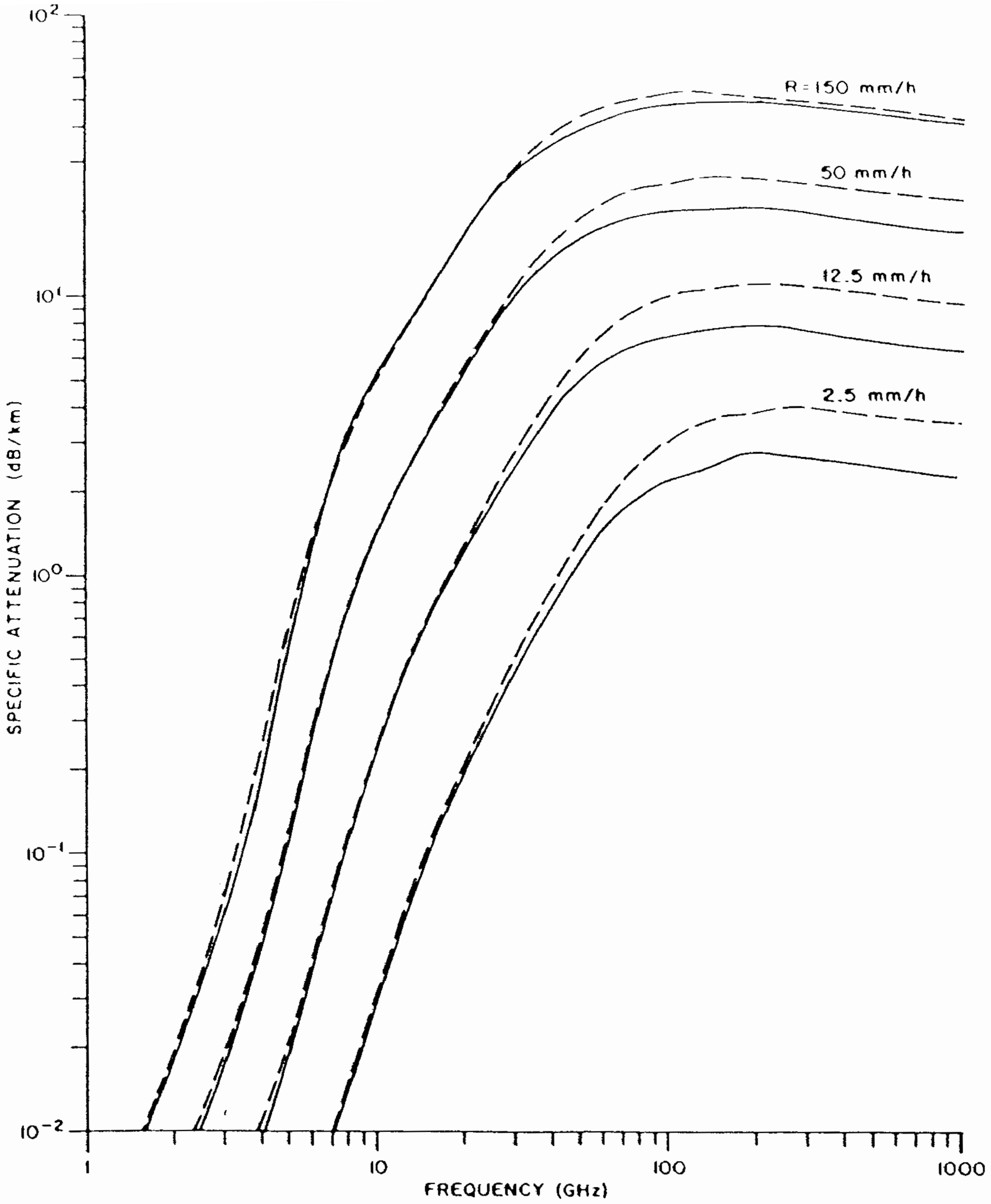


Fig. 32. Frequency characteristics of rain attenuation at rain temperature of 20°C, for the LP (solid curves) and MP (dashed curves) drop-size distributions. (After Rogers and Olsen [159].)

LAWS & PARSONS

MARSHALL & PALMER

## RAINDROP CANTING

RAINDROPS DON'T ALWAYS FALL STRAIGHT DOWN VERTICALLY;

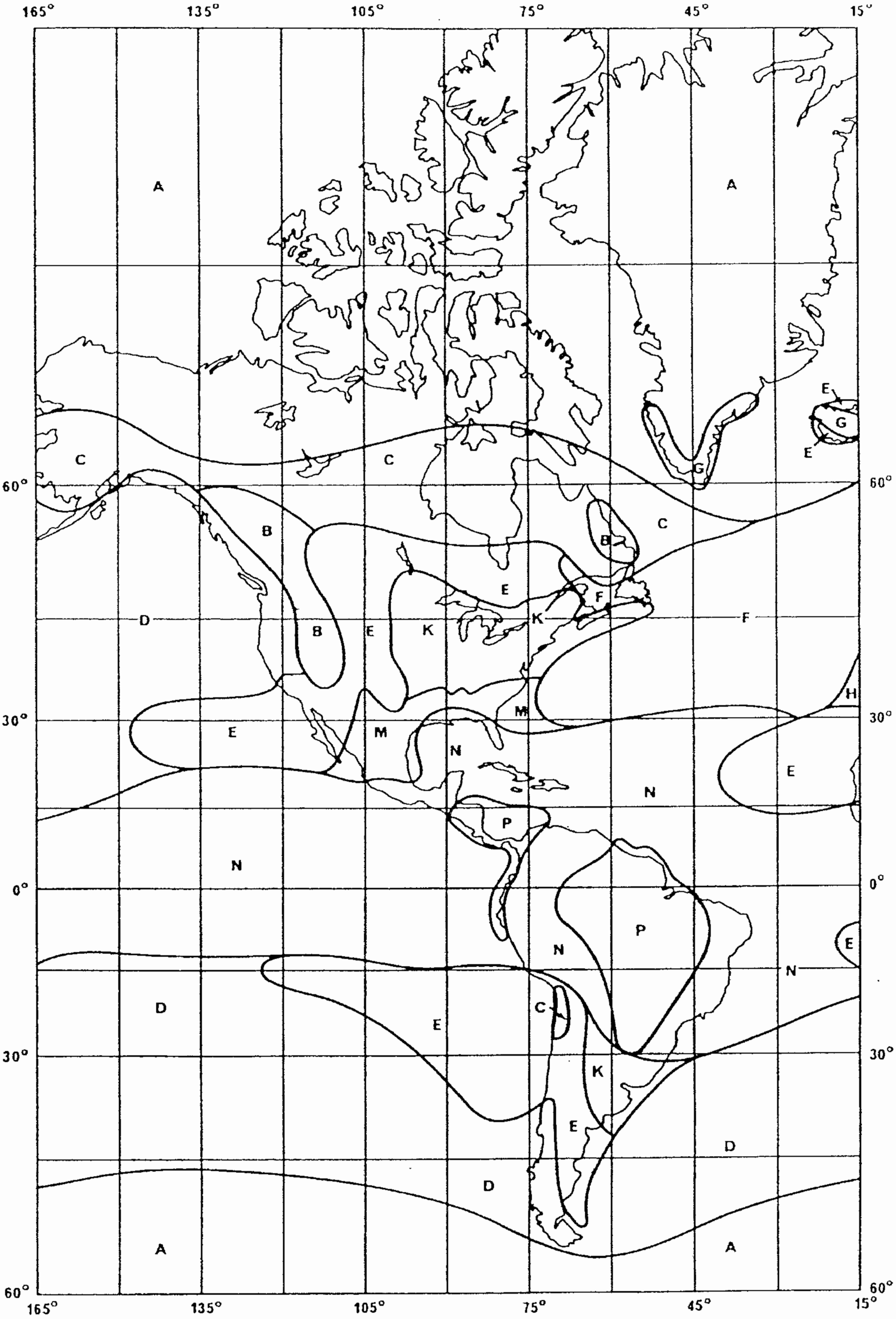
SMALL ENOUGH TO IGNORE EXCEPT JUST BELOW THE MELTING LAYER

## SNOW & HAIL

SNOW DOESN'T CAUSE APPRECIABLE ATTENUATION UNLESS THE FLAKES ARE WET.

HAIL DOESN'T CAUSE SIGNIFICANT ATTENUATION UNLESS THE HAIL STONES ARE WET-

- HAIL IS RELATIVELY INFREQUENT.


165°
135°
105°
75°
45°
15°
60°
30°
0°
30°
60°
A
A
C
C
C
B
B
B
D
E
K
E
K
F
F
G
G
E
E
E
M
M
N
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P
P
H
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C
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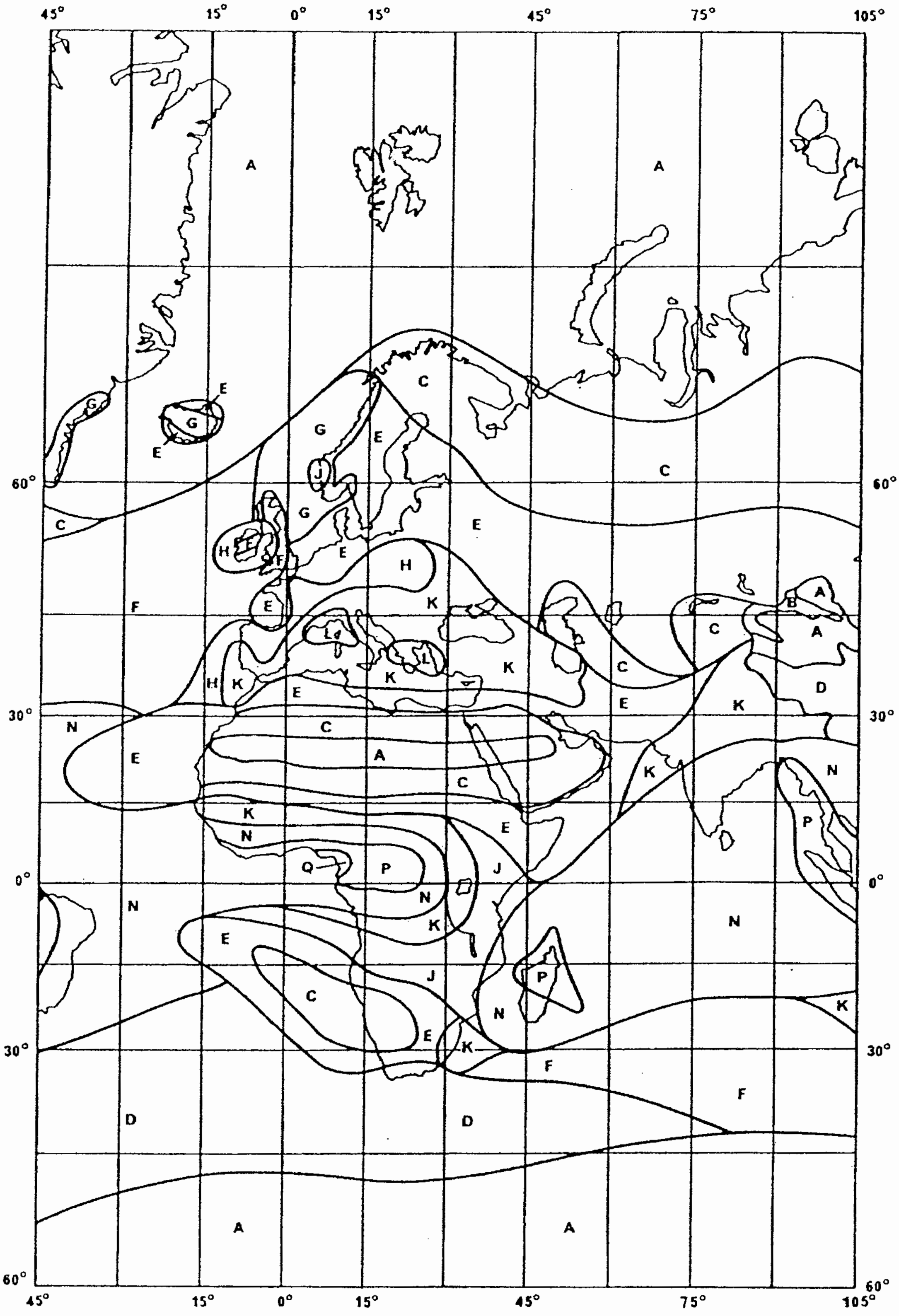
D
D
A
A


45°
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75°
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A
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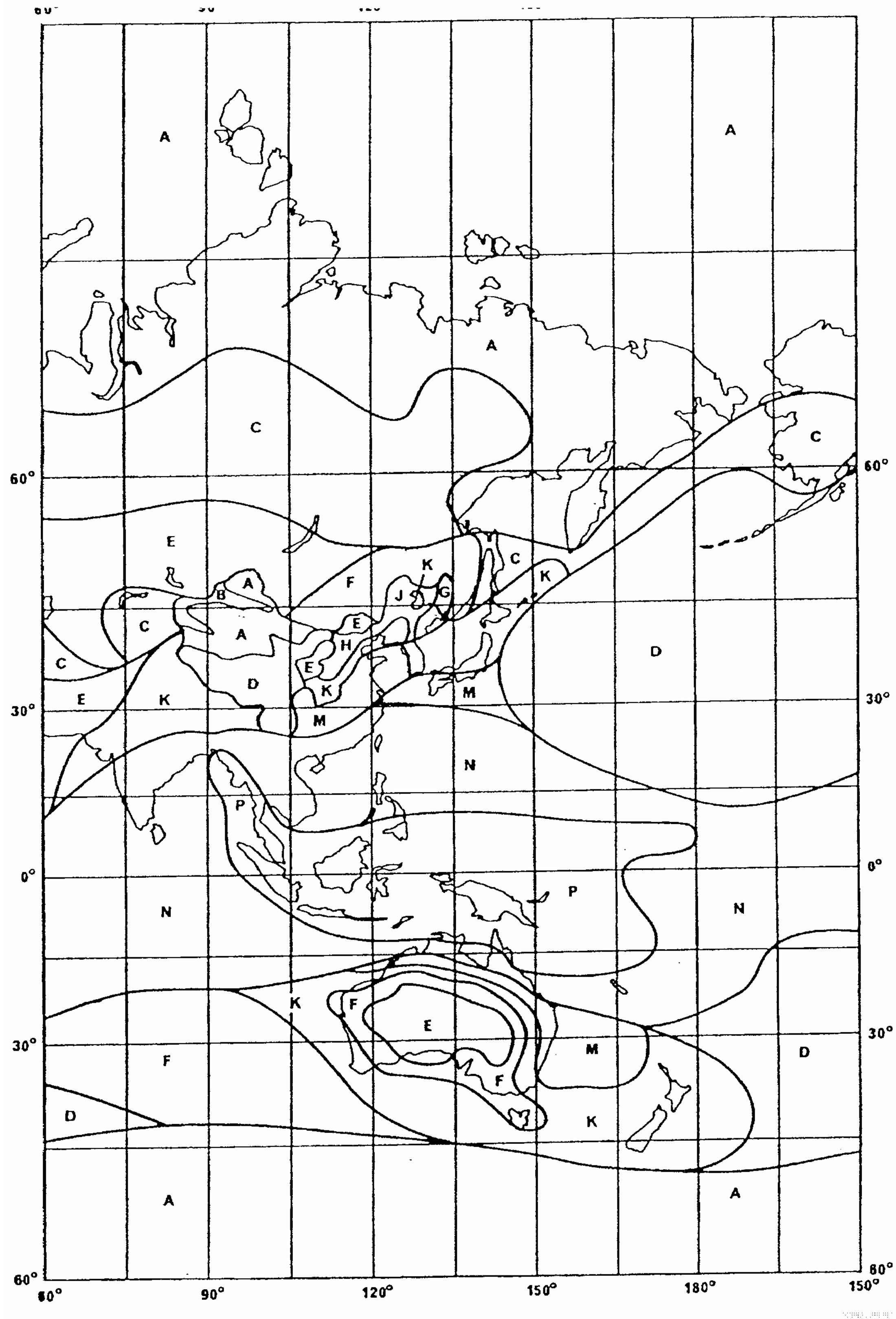
D
A
A

D02-sc


A
A
A
C
C
60°
60°
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K
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B
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D
E
D
K
E
K
M
30°
30°
M
N
P
0°
0°
P
N
N
K
F
E
M
30°
30°
F
D
F
D
K
A
A
60°
60°
60°
90°
120°
150°
180°
150°

D03-sc

TABLE 1

## Rain climatic zones

**Rainfall intensity exceeded (mm/h)** (Reference to Figs. 1 to 3)

| Percentage of time (%) | A | B | C | D | E | F | G | H | J | K | L | M | N | P | Q |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | <0.1 | 0.5 | 0.7 | 2.1 | 0.6 | 1.7 | 3 | 2 | 8 | 1.5 | 2 | 4 | 5 | 12 | 24 |
| 0.3 | 0.8 | 2 | 2.8 | 4.5 | 2.4 | 4.5 | 7 | 4 | 13 | 4.2 | 7 | 11 | 15 | 34 | 49 |
| 0.1 | 2 | 3 | 5 | 8 | 6 | 8 | 12 | 10 | 20 | 12 | 15 | 22 | 35 | 65 | 72 |
| 0.03 | 5 | 6 | 9 | 13 | 12 | 15 | 20 | 18 | 28 | 23 | 33 | 40 | 65 | 105 | 96 |
| 0.01 | 8 | 12 | 15 | 19 | 22 | 28 | 30 | 32 | 35 | 42 | 60 | 63 | 95 | 145 | 115 |
| 0.003 | 14 | 21 | 26 | 29 | 41 | 54 | 45 | 55 | 45 | 70 | 105 | 95 | 140 | 200 | 142 |
| 0.001 | 22 | 32 | 42 | 42 | 70 | 78 | 65 | 83 | 55 | 100 | 150 | 120 | 180 | 250 | 170 |