BANDLIMITED MODULATED DIGITAL COMMUNICATIONS ①

THE IDEAL CHANNEL

FOR DISTORTIONLESS TRANSMISSION WE REQUIRE:

* CONSTANT MAGNITUDE RESPONSE
* LINEAR PHASE (THAT IS THE PHASE SHIFT MUST BE LINEAR WITH FREQUENCY)

TIME DELAY, t_0 (SECONDS)

$$t_0 = \frac{\theta \leftarrow \text{PHASE, RADIANS}}{2\pi f \leftarrow \text{FREQUENCY, Hz.}}$$

FOR CONSTANT TIME DELAY $\Rightarrow \theta \propto f$.

NOTE: IN REALITY WE CANNOT ACHIEVE ANYTHING LIKE THIS WITHOUT EQUALIZATION

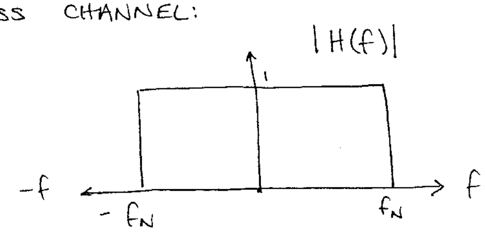
PHASE OR AMPLITUDE CORRECTION IS TERMED EQUALIZATION.

IF THE CHANNEL RESPONSE IS UNKNOWN OR VARIABLE WE CAN USE ADAPTIVE EQUALIZATION

②

THE IDEAL LOW-PASS CHANNEL

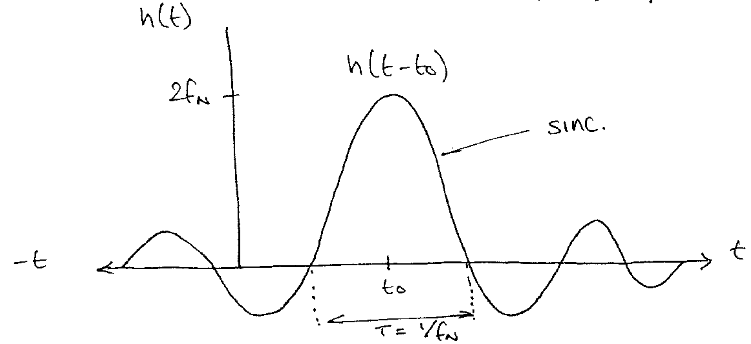
THE FREQUENCY RESPONSE OF AN IDEAL LOW-PASS CHANNEL:



IMPULSE RESPONSE:

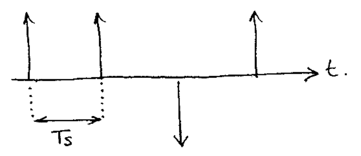
$$h(t) = \int_{-\infty}^{\infty} H(f)\, e^{j2\pi f t}\, df.$$

$$= 2 f_N \frac{\sin[2\pi f_N (t-t_0)]}{2\pi f_N (t-t_0)}$$



NON CAUSAL => NOT REALIZABLE.

BASEBAND SIGNALLING



FROM NYQUIST FOR ZERO ISI;

$$T_S = \frac{1}{2f_N}.$$

INTER SYMBOL INTERFERENCE (ISI)

A BAND LIMITED CHANNEL DISPERSES (SPREADS) A PULSED WAVEFORM PASSING THROUGH IT.

CHANNEL BW $\gg$ PULSE BW $\Rightarrow$ SLIGHT SPREAD

CHANNEL BW $\simeq$ PULSE BW $\Rightarrow$ LARGE SPREAD

IF SPREADING EXCEEDS THE SYMBOL DURATION, SYMBOLS OVERLAP $\Rightarrow$ I.S.I.

* ISI INCREASES ERROR RATE.

* INCREASING SNR DOESN'T HELP.

— PIONEERING WORK DONE BY HARRY NYQUIST.

SO, FOR ZERO ISI; $T_S = \frac{1}{2f_N}$.

THAT IS $f_S = 2f_N$, WHERE f_S IS THE SYMBOL RATE

FOR THE CASE OF BINARY COMMUNICATIONS, THE SYMBOL RATE f_S IS EQUAL TO THE BIT RATE. i.e

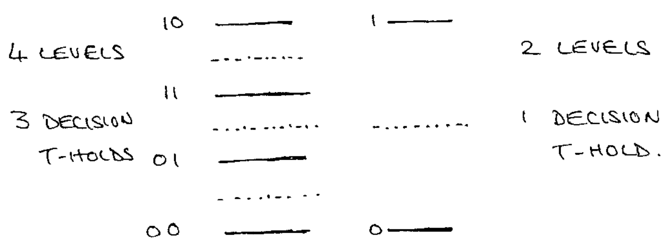
$$f_B = f_S$$

FOR MULTI-LEVEL SIGNALLING (M-ary, M-LEVELS)

$$f_B = f_S \log_2 M.$$

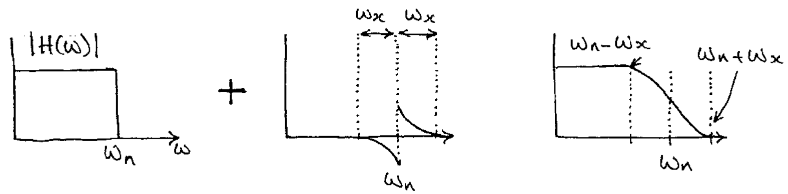
NOTE: NOISE PERFORMANCE IS BEST FOR BINARY SYSTEMS.

MULTI-LEVEL SYSTEMS REQUIRE AN EXTRA 6dB OF S/N RATIO EVERY TIME 'M' IS DOUBLED IN ORDER TO MAINTAIN THE SAME B.E.R.



BITS PER SYMBOL	M - LEVELS
1	2
2	4
3	8
4	16
⋮	⋮
ETC.	

SINUSOIDAL SPECTRUM SHAPING



* ALLOWS FOR A MORE REALISTIC CHANNEL (AND PULSE) RESPONSE WHILE MAINTAINING THE PROPERTY OF ZERO ISI.

THE ROLL-OFF FACTOR α IS DEFINED AS;

$$\alpha = \frac{\omega_x}{\omega_n}$$

THE SHAPED CHANNEL RESPONSE IS GIVEN BY;

$$|H(\omega)| = \begin{cases} 1 & \omega < (\omega_n - \omega_x) \\ \cos^2\left[\frac{\pi}{4\alpha\omega_n}(\omega - \omega_n(1-\alpha))\right] & (\omega_n - \omega_x) \leq \omega \leq (\omega_n + \omega_x) \\ 0 & \omega > (\omega_n + \omega_x) \end{cases}$$

THEREFORE;

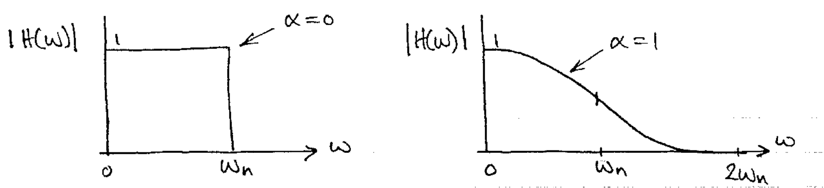
$\alpha = 0$ IDEAL CHANNEL RESPONSE
$\alpha = 1$ FULLY RAISED COSINE RESPONSE

THE CHANNEL BANDWIDTH NOW BECOMES:

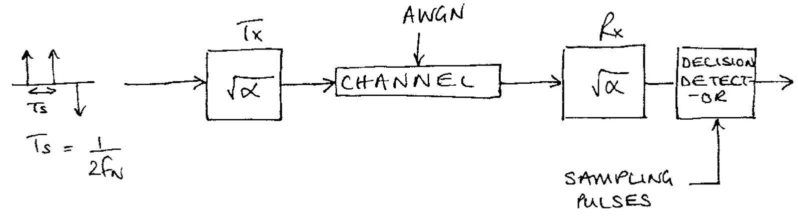
$$\begin{aligned} BW &= \omega_n + \omega_x \\ &= \omega_n + \alpha\,\omega_n \\ &= \omega_n(1+\alpha) \end{aligned}$$ FOR ZERO ISI

$f_N = \frac{f_s}{2}$ SO THE BASEBAND BANDWIDTH IS;

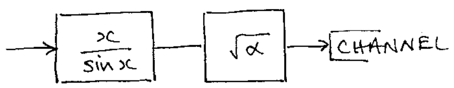
$$\boxed{BW_{Hz} = \frac{f_s}{2}(1+\alpha)}$$



OPTIMUM P_E PERFORMANCE FOR AWGN (ADDITIVE WHITE GAUSSIAN NOISE) AND CHANELS WITH A FLAT MAGNITUDE RESPONSE IS OBTAINED USING POLAR BASE BAND SIGNALS AND PARTIONING $H(f)$ SUCH THAT



FOR FULLWIDTH SIGNALS ;



FOR WIDEBAND CHANNELS SAME RESULT CAN BE OBTAINED WITH AN INTEGRATE & DUMP MATHED FILTER DETECTOR

$$\frac{x}{\sin x} = \frac{\omega T_s/2}{\sin \omega T_s/2} = \frac{\pi f T_s}{\sin(\pi f T_s)}$$

FOR POLAR BASEBAND SIGNALLING;

$$P_E = Q\left[\sqrt{\frac{2E_b}{N_o}}\right]$$

N_o – ONE SIDED NOISE SPECTRAL DENSITY MEASURED AT THE RECEIVER.

MODULATED SIGNALS

OPTIMUM PERFORMANCE IS OBTAINED WITH COHERENT DETECTION OF PSK (PHASE SHIFT KEYING), WHICH IS IN EFFECT DSB-SC AM WITH A POLAR MODULATING SIGNAL;

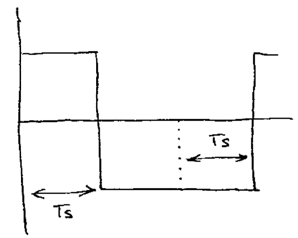
TRANSMITTED SIGNAL $= s(t)\, A_o \cos(w_c t)$

w_c – CARRIER FREQUENCY

$$s(t) = \begin{cases} +1 \ (\text{binary '1'}) & \text{ie} \quad + A_o \cos(w_c t) \\ -1 \ (\text{binary '0'}) & \quad\;\; - A_o \cos(w_c t) \end{cases}$$

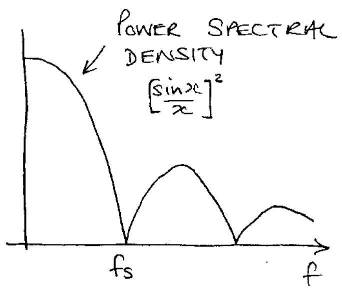
NOTE : THIS IS A CONSTANT ENVELOPE SIGNAL (THERFORE CAPABLE OF BEING AMPLIFIED BY NON-LINEAR POWER AMPLIFIERS).
BANDWIDTH = 2x BASE-BAND BANDWIDTH

FOR NRZ (NON-RETURN-TO-ZERO) RECTANGULAR MODULATING SIGNALS;



$\Rightarrow$

$$\frac{1}{2f_s} = T_s$$



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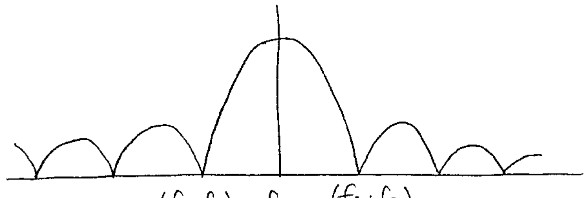
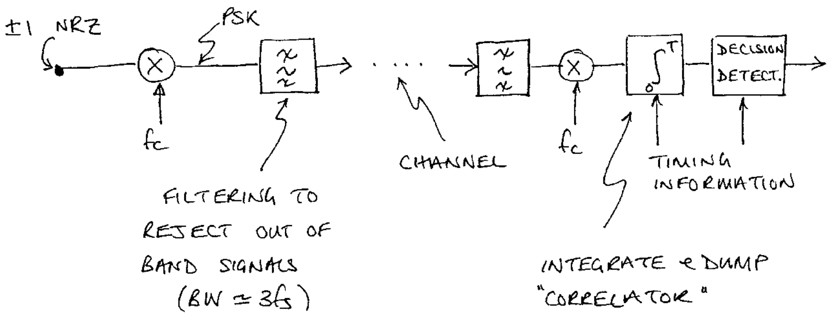
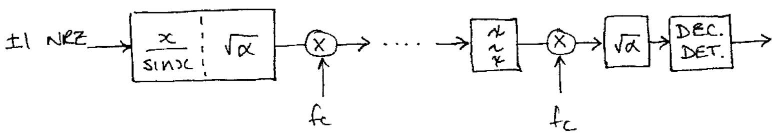
HENCE THE PSK SPECTRUM;





FOR MINIUMUM BANDWIDTH SYSTEMS



$$\text{BANDWIDTH} = 2 \times \left[\frac{f_s}{2} (1+\alpha) \right]$$

IN THIS CASE, SIGNAL AT Tx. NO LONGER A CONSTANT ENVELOPE, DUE TO THE SHAPING.

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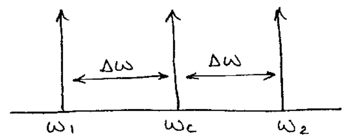
FOR NON-CONSTANT ENVELOPE SIGNALS, THE POWER AMPLIFIERS MUST BE LINEAR.

LINEAR POWER AMPLIFIERS => $ £!

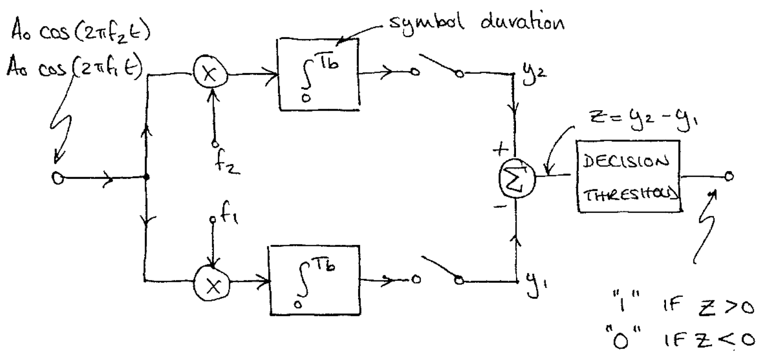
FREQUENCY SHIFT KEYING

BINARY "1" => $A_0 \cos(\omega_2 t) = A_0 \cos[(\omega_c + \Delta\omega)t]$

BINARY "0" => $A_0 \cos(\omega_1 t) = A_0 \cos[(\omega_c - \Delta\omega)t]$



COHERENT (OR SYNCHRONOUS) DETECTION OF FSK SIGNALS CORRELATOR



SUPPOSE $A_0 \cos(\omega_2 t)$ IS INPUT INTO THE COHERENT DEMODULATOR:

WE REQUIRE A +1 AS THE BINARY OUTPUT, WHICH MEANS A ZERO FROM THE BOTTOM (ω_1) OUTPUT.

HENCE;

$$y_1 = \int_0^{T_b} \cos(\omega_1 t)\cos(\omega_2 t)\,dt.$$

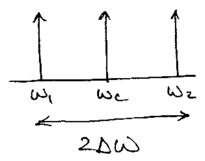
[SINCE $\cos A \cos B = \frac{1}{2}\cos(A+B) + \frac{1}{2}\cos(A-B)$

$$\Rightarrow \quad y_1 = \frac{1}{2}\int_0^{T_b} \underbrace{\cos[(\omega_1+\omega_2)t]}_{\substack{\text{IF WE LOW-PASS FILTER} \\ \text{WE CAN NEGLECT THIS}}} + \cos[(\omega_2-\omega_1)t]\,dt.$$

HENCE WE HAVE;

$$y_1 = \frac{1}{2}\int_0^{T_b} \cos[(\omega_2-\omega_1)t]\,dt.$$

$$y_1 = \frac{1}{2}\int_0^{T_b} \cos(2\Delta\omega t)\,dt.$$



$$y_1 = \frac{1}{2}\int_0^{T_b} \cos(2\Delta\omega t)\,dt$$

INTEGRATING, WE HAVE;

$$y_1 = \frac{1}{2}\cdot\frac{1}{2\Delta\omega}\sin(2\Delta\omega t)\Big|_0^{T_b}$$

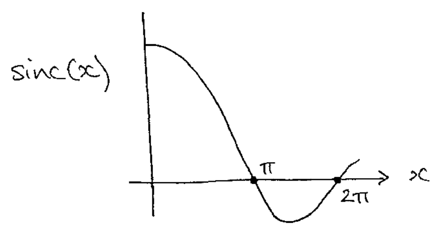
$$= \frac{1}{2}\cdot\frac{\sin(2\Delta\omega T_b)}{2\Delta\omega}$$

MULTIPLYING TOP AND BOTTOM BY T_b.

$$\Rightarrow \quad y_1 = \frac{T_b}{2}\,\frac{\sin(2\Delta\omega T_b)}{2\Delta\omega T_b}$$

$$= \frac{T_b}{2}\,\mathrm{sinc}(2\Delta\omega T_b)$$

THE FIRST ZERO OF THE 'SINC' FUNCTION OCCURS WHEN $x = \pi$, $\mathrm{sinc}(\pi) = 0$.



FOR $y_1 = 0$

$$\Rightarrow 2\Delta\omega T_b = \pi$$

$$\Rightarrow \Delta\omega = \frac{\pi}{2T_b}$$

OR

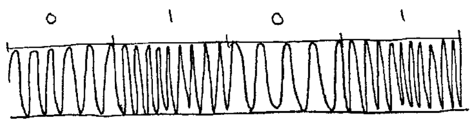
$$\Delta f = \frac{1}{4T_b}$$

SINCE $\Delta f = \frac{1}{4T_b}$, $f_2 - f_1 = \frac{f_b}{2} = \frac{1}{2T_b}$

THIS IS THE <u>MINIMUM FREQUENCY SPACING</u> FOR THE TWO SIGNALS TO BE <u>ORTHOGONAL</u>

<u>FOR EXAMPLE</u>

SUPPOSE WE HAVE A BIT RATE OF 1200 BITS/SEC. CARRIER FREQUENCY, $f_c = 1500\ Hz$.



$\Delta f = \frac{1}{4T_b}$, $T_b = \frac{1}{f_c}$ $\Rightarrow \Delta f = \frac{1500}{4 \cdot 1} = 300\ Hz$.

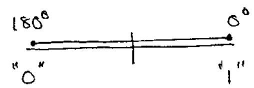
$\Rightarrow f_1 = f_c - \Delta f = 1500 - 300 = 1200\ Hz$
$\Rightarrow f_2 = f_c + \Delta f = 1500 + 300 = 1800\ Hz$.

<u>QUADRATURE MODULATION</u>

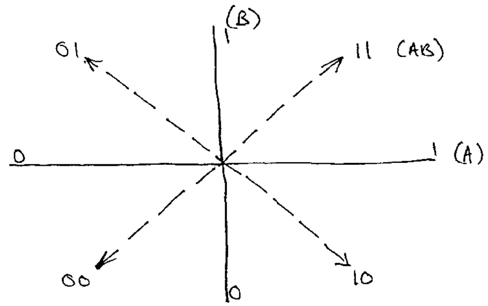
<u>QPSK (QUADRATURE PHASE SHIFT KEYING)</u>

EQUIVALENT TO TWO SIMULTANEOUS <u>BINARY</u> PSK CHANNELS IN QUADRATURE AND AT HALF RATE.

PHASOR DIAGRAM OF PSK;



PHASOR DIAGRAM OF QPSK

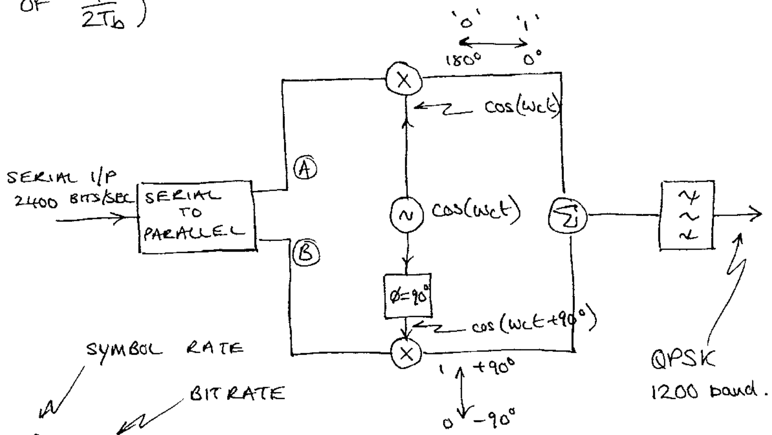


NOTE: THE SEQUENCE OF SYMBOLS AS THE PHASE IN CONTINUALLY CHANGED IS IN A CYCLIC <u>GRAY</u> CODE. ONLY ONE BIT CHANGES <u>AT</u> A TIME SO A PHASE CHANGE ERROR NORMALLY ONLY PRODUCES A ONE BIT ERROR

NOTE : BECAUSE QPSK GIVES A CONSTANT ENVELOPE SIGNAL WITH PHASE CHANGES $\mp 90^\circ$ AND 180° WE CAN USE HIGH POWER <u>NON-LINEAR</u> AMPS.

GENERATION OF QPSK

INPUT DATA STREAM IS SPLIT INTO TWO HALF-RATE STREAMS (AT A SYMBOL RATE OF $\frac{1}{2T_b}$)



SYMBOL RATE

BIT RATE

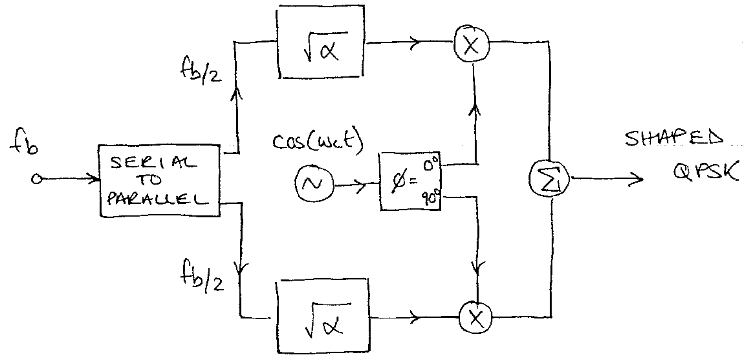
$f_s = f_b/2$

⇒ HALF RATE, <u>BUT</u> TWO BITS PER SYMBOL.

IF PRE-MODULATION (BASEBAND) FILTERING IS USED FOR SPECTRAL SHAPING, THE SIGNAL WILL <u>NOT</u> HAVE A CONSTANT AMPLITUDE

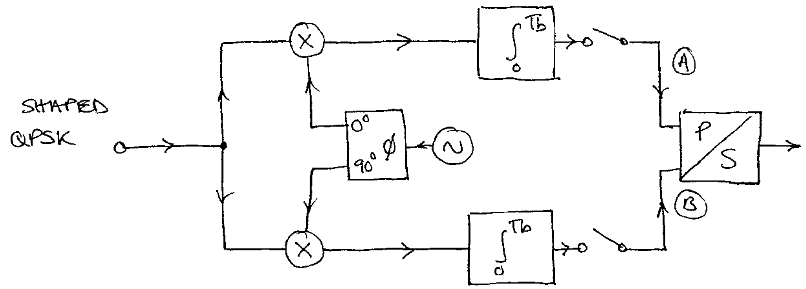
* IT BECOMES MORE LIKE QUAD. A.M., BUT QPSK IS STILL OFTEN USED TO DESCRIBE IT.

WITH SPECTRAL SHAPING;



NOTE : BECAUSE OF THE SHAPING, WE HAVE TO USE <u>LINEAR</u> P.A STAGES.

DETECTION OF QPSK



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INPUT PHASE	BINARY CODE	A 0°	B 90°
+45°	1 1	+	+
−45°	1 0	+	−
+135°	0 1	−	+
−135°	0 0	−	−

WITH PREMOD FILTERING $BW = f_s(1+\alpha) = \frac{f_B}{2}(1+\alpha)$

GENERATION OF QPSK (CONTINUED)

THE TWO DATA STREAMS CONSIST OF a_I (EVEN) AND a_Q (ODD) BITS.

THE IN-PHASE (I) AND QUADRATURE (Q) CARRIERS CAN BE DESCRIBED AS

$$I : \frac{1}{\sqrt{2}} \cos(\omega_c t + \pi/4)$$

AND

$$Q : \frac{1}{\sqrt{2}} \sin(\omega_c t + \pi/4)$$

[NOTE: THE $\frac{1}{\sqrt{2}}$ AND $\pi/4$ ARE JUST FOR CONVENIENCE]

THE SIGNAL $s(t)$ IS THEN GIVEN BY;

$$s(t) = \frac{1}{\sqrt{2}} a_I(t)\cos(\omega_c t + \pi/4) + \frac{1}{\sqrt{2}} a_Q(t)\sin(\omega_c t + \pi/4)$$

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WITH A LITTLE JIGGERY-POKERY WE CAN REDUCE THIS TO:

$$s(t) = \cos[\omega_c t + \theta(t)]$$

WHERE $\theta(t) = 0°, \mp 90°$ OR $180°$ CORRESPONDING TO THE FOUR COMBINATIONS OF $a_I(t)$ AND $a_Q(t)$.