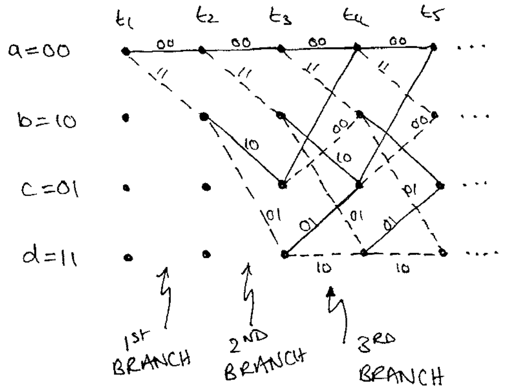
## VITERBI DECODING

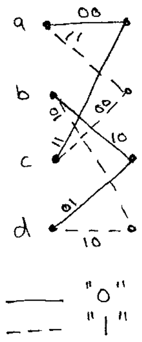
WE CAN DECODE OUR CONVOLUTIONAL CODE BY CHOOSING THE PATH THROUGH THE CODE THAT HAS THE LARGEST METRIC

- IN OUR CASE, GIVEN THAT WE ARE CONSIDERING A BINARY SYMMETRIC CHANNEL (BSC) MEANS CHOOSING THE PATH THAT GIVES THE SMALLEST HAMMING DISTANCE FROM THE RECEIVED SIGNAL

## HOW THE VITERBI DECODING ALGORITHM WORKS:

FOR THE ½ RATE K=3 CODE WE HAVE BEEN LOOKING AT WE CAN SEE THAT AT BRANCH LEVEL $j=3$ (AND SUBSEQUENTLY) ON THE TRELLIS THERE ARE TWO PATHS INTO EACH NODE;





THE DECODER DECIDES WHICH OF THESE TWO PATHS TO RETAIN. THE RETAINED PATH IS CALLED THE SURVIVOR.

THE DECISION IS MADE BY CHOOSING THE PATH WITH THE SMALLEST HAMMING DISTANCE BETWEEN THE CODED SEQUENCE FOR THE PATH AND THE RECEIVED SEQUENCE - THE OTHER PATH IS DISCARDED.

## REMINDER: HAMMING DISTANCE

CONSIDER TWO VECTORS $u, v$ THE HAMMING DISTANCE $d(u,v)$ IS DEFINED TO BE THE NUMBER OF ELEMENTS IN WHICH THEY DIFFER. FOR EXAMPLE;

$$u = [1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1]$$

$$v = [0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 0]$$

$$d(u,v) = 6$$

BY THE WONDERS OF MOD-2 ADDITION IF WE ADD $u \oplus v$ WE HAVE A ONE IN THE POSITIONS WHERE THE VECTORS DIFFER

$u+v = [1\ 1\ 1\ 0\ 1\ 1\ 0\ 0\ 1]$ $\quad w(u \oplus v) = d(u,v) = 6$

THE HAMMING WEIGHT OF $u \oplus v$ IS THE HAMMING DISTANCE BETWEEN $u$ AND $v$

SO WE WORK THROUGH THE TRELLIS DISCARDING PATHS THAT HAVE THE LARGEST HAMMING DISTANCE WHEN TWO PATHS MERGE.

IF THE TWO PATHS HAVE THE SAME PATH METRIC THE DECODER HAS TO MAKE A GUESS!

AT EACH TIME $t_i$ THERE ARE $2^{K-1}$ STATES IN THE TRELLIS. HENCE FOR $K=3$ THERE WILL BE NO MORE THAN $2^{K-1}=4$ SURVIVOR PATHS FOR WHICH WE NEED TO STORE THE HAMMING DISTANCES.

AT EACH TIME $t_i$ WE CAN LABEL EACH TRELLIS BRANCH WITH THE HAMMING DISTANCE BETWEEN THE RECEIVED CODEWORD AND THE CORRESPONDING BRANCH WORD AT THE ENCODER.

SUPPOSE OUR MESSAGE SEQUENCE M IS;

$M = \quad 1 \quad 1 \quad 0 \quad 1 \quad 1$

$C_M = \quad 11 \quad 01 \quad 01 \quad 00 \quad 01$ ← CODED SEQUENCE

$R = \quad 11 \quad 01 \quad 01 \quad 10 \quad 01$ ← RECEIVED SEQUENCE

SINGLE BIT ERROR ↑

HENCE SINCE AT $t_6$ THE PATH WITH THE SMALLEST METRIC IS THE $\lambda d=1$ PATH, THE STATE TRANSITIONS ARE

ⓐ → ⓑ → ⓓ → ⓒ → ⓑ → ⓓ

HENCE THE CORRECTED CODE SEQUENCE IS;

11 01 01 00 01

AND THE MESSAGE IS;

1 1 0 1 1

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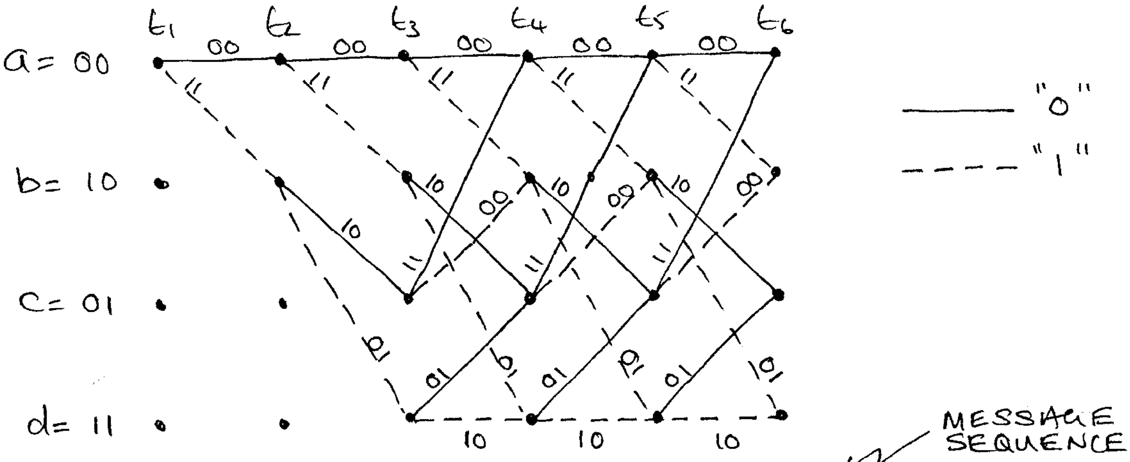
IDEALLY THE DECODING PROCESS SHOULD BE CONTINUED FOR THE WHOLE OF THE MESSAGE SEQUENCE LENGTH (SAY L BITS), AND THEN THE MOST LIKELY PATH CHOSEN

BUT TO AVOID AN EXCESSIVE DELAY IN DECODING AND TO AVOID HAVING TO HAVE A LARGE PATH MEMORY, IT HAS BEEN SHOWN THAT IF THE DECODER OPERATES WITH A PATH MEMORY OF 4 OR 5 TIMES THE CONSTRAINT LENGTH K, THE PERFORMANCE IS CLOSE TO OPTIMUM.

MOST PRACTICAL DECODER HAVE A FIXED PATH LENGTH MEMORY – WHEN THE MEMORY IS FULL A DECODING DECISION IS FORCED AT THAT LEVEL OF THE TRELLIS.

## ENCODER TRELLIS

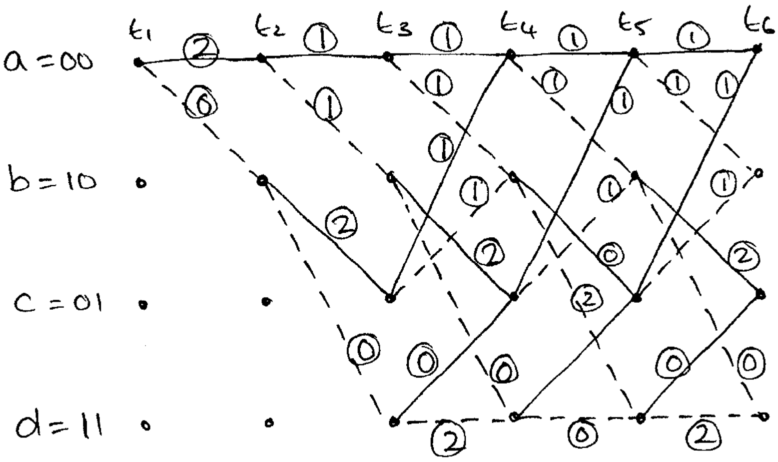


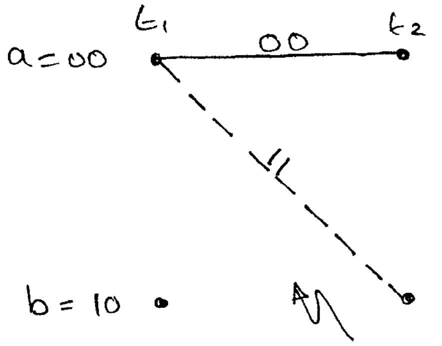
M = 1 1 0 1 1 ... ← MESSAGE SEQUENCE

$C_M$ = 11 01 01 00 01 ... ← CODED SEQUENCE

R = 11 01 01 10 01 ... ← RECEIVED SEQUENCE

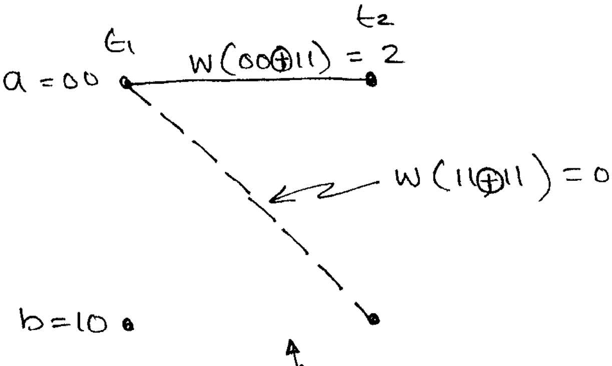
## DECODER TRELLIS



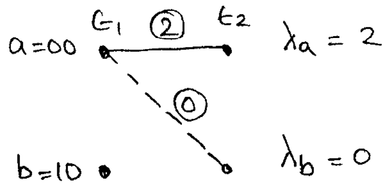


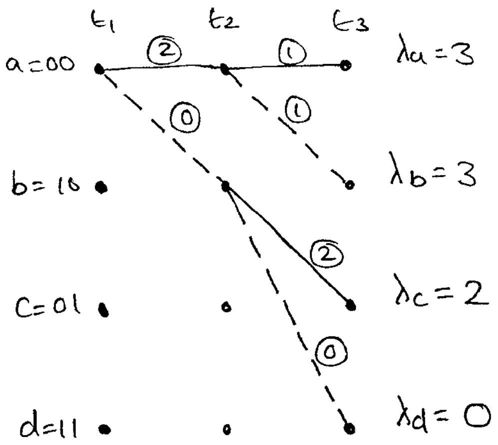
R = 11

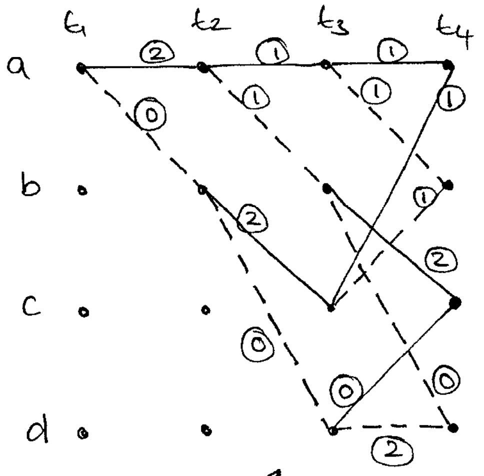
ENCODER
TRELLIS
FRAGMENT

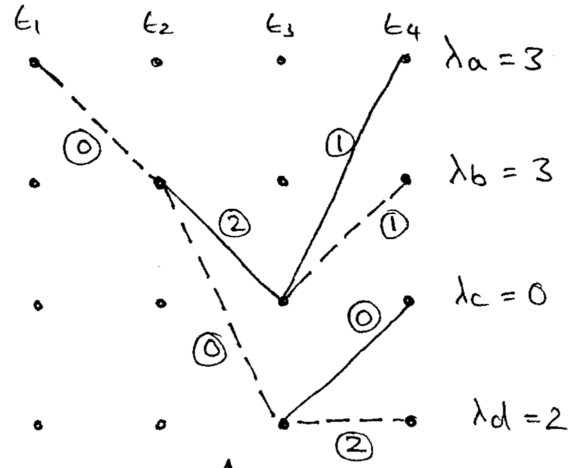


DECODER
TRELLIS
FRAGMENT



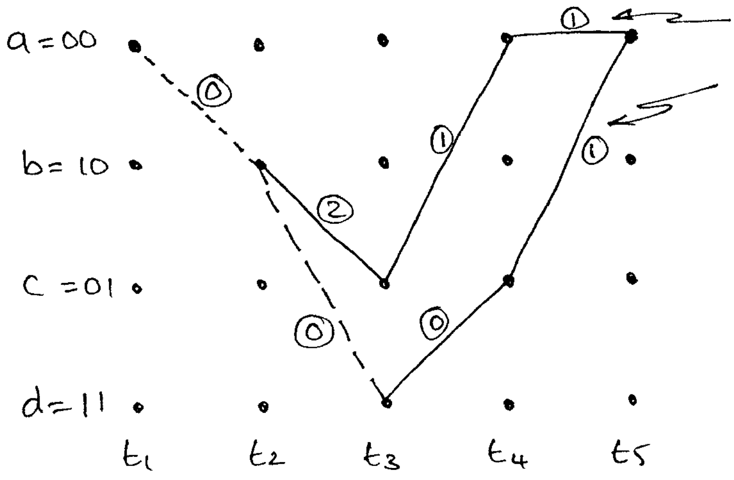






AT $t_4$, EACH STATE HAS TWO PATHS, WE CAN COMPARE METRICS AND DISCARD ONE

HENCE THE SURVIVING PATHS ARE SHOWN ABOVE

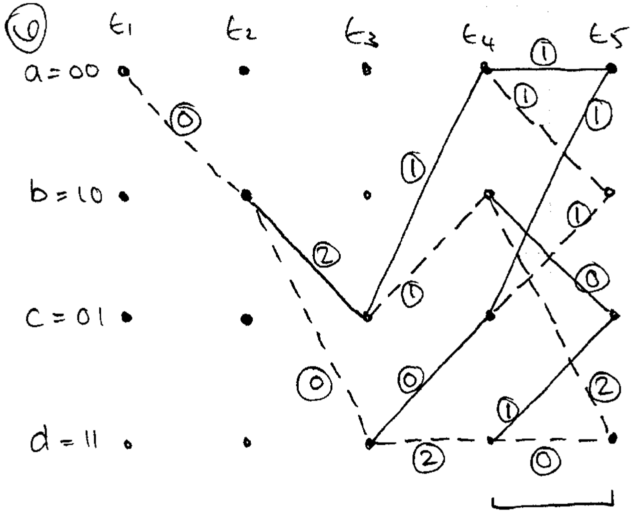


PATH METRIC = 4

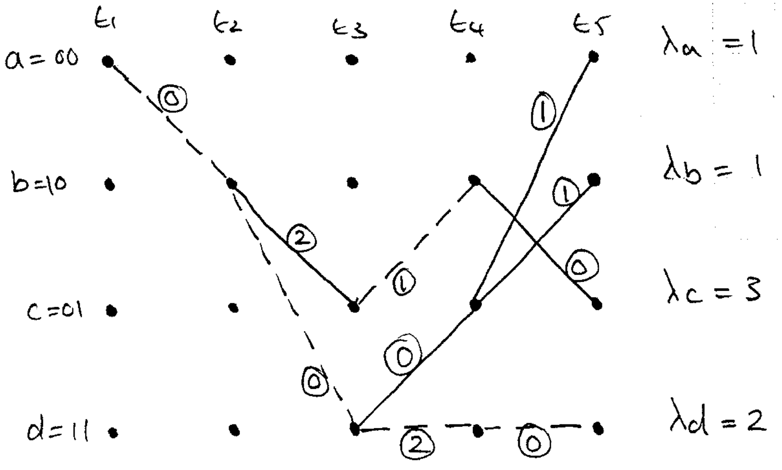
PATH METRIC = 1

⇒ WE KEEP THE PATH WITH THE SMALLEST METRIC

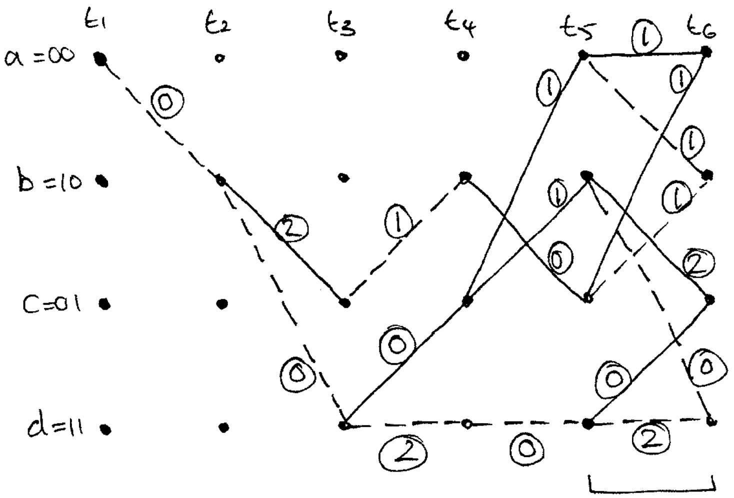
⑥



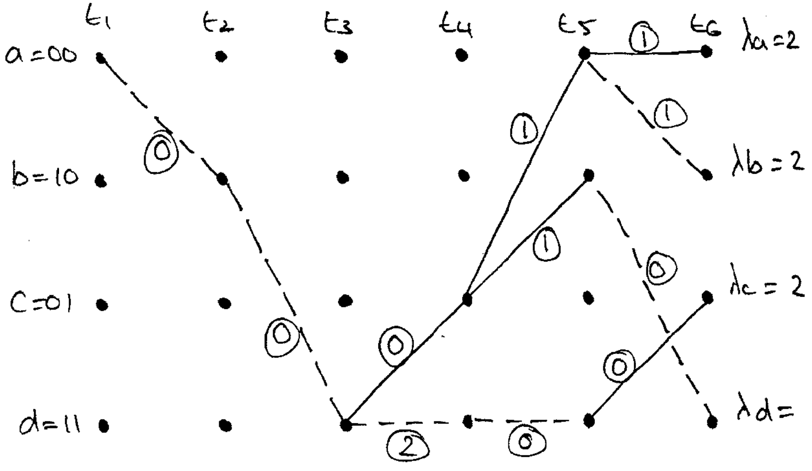
ADD 4th BRANCH



EVALUATE SURVIVORS AT t5



ADD 5th BRANCH



EVALUATE SURVIVORS AT t5