CONVOLUTIONAL CODES

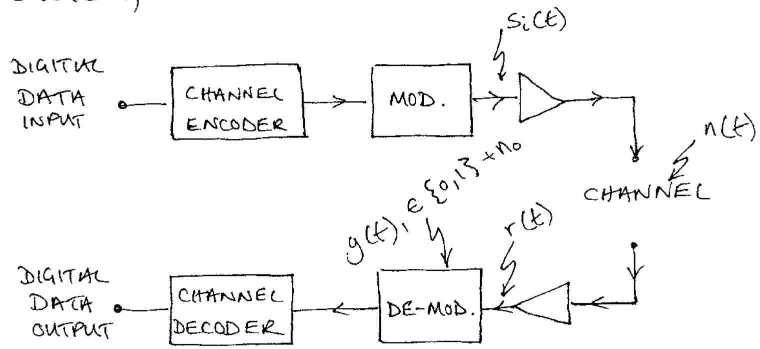
* DEFINITIONS : n, k, K.
* IMPLEMENTATION OF CONVOLUTIONAL CODERS
* REPRESENTATION
  - STATE TABLE.
  - STATE DIAGRAM.
  - TREE DIAGRAM.
  - TRELLIS DIAGRAM.
* DECODING OF CONVOLUTIONAL CODES
  - MAXIMUM LIKELIHOOD DECODING.
  - VITERBI DECODING.

THIS LECTURE ...

* BEFORE WE LOOK AT DECODING IN DETAIL, LET'S TAKE ANOTHER LOOK AT THE CHANNEL MODEL ...

CHANNEL MODELS: HARD AND SOFT DECISIONS

CONSIDER AGAIN OUR DIGITAL COMMUNICATIONS SYSTEM;



IT IS THE JOB OF THE MODULATOR AND DEMODULATOR TO TURN THE BITSTREAM INTO ANALOGUE SIGNALS AND BACK.

INTUITIVELY, WE CAN SEE THAT IF THE DEMODULATOR GETS IT WRONG BECAUSE THE SIGNAL IS MARGINAL, WE ARE INTRODUCING ERRORS.

SUPPOSE WE SEND A BINARY SIGNAL INTO THE CHANNEL, REPRESENTED AS "s_1" FOR A BINARY ONE, AND AS "s_0" FOR A BINARY ZERO.

THE SIGNAL AT THE INPUT OF THE DEMODULATOR, FOLLOWING THE ADDITION OF A NOISE SIGNAL $n(t)$ IS;

$$r(t) = s_i(t) + n(t)$$

WHERE $n(t)$ IS A ZERO-MEAN GAUSSIAN PROCESS.

THE DEMODULATOR IS FACED WITH THE TASK OF TAKING THE SIGNAL $r(t)$ AND PRODUCING JUST A SINGLE NUMBER $y(t)$; (EITHER A "0" OR A "1")

SINCE THE RECEIVED, MODULATED SIGNAL IS AFFECTED BY NOISE (FROM THE CHANNEL) IT FOLLOWS THAT;

$$y(t) = a_i + n_o$$

WHERE; a_i IS THE SIGNAL COMPONENT, AND
n_o IS A ZERO-MEAN GAUSSIAN RANDOM VARIABLE

[THIS MEANS THAT $y(t)$ IS ALSO A GAUSSIAN R.V. WITH A MEAN OF a_1 OR a_2.]

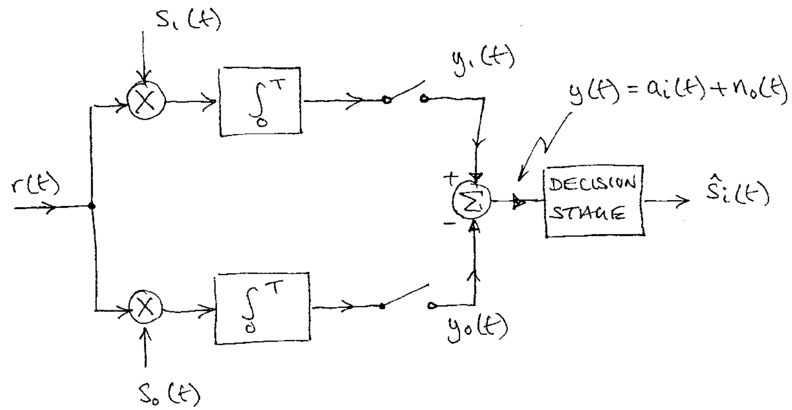
THE SECOND TASK OF THE DE-MODULATOR IS THAT OF DECISION MAKING. THAT IS GIVEN $y(t)$ WE HAVE DECIDE WHETHER WE HAVE A "1" OR "0"

WE CAN EITHER MAKE A <u>HARD</u> DECISION

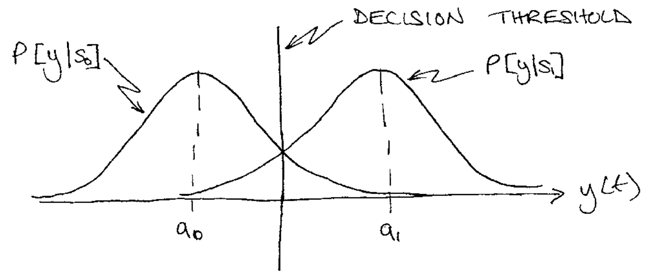
OR A <u>SOFT</u> DECISION ...

<u>HARD (OR FIRM) DECISIONS</u>

IN THE CASE OF A HARD DECISION, THE OUTPUT OF THE DEMODULATOR IS QUANTIZED INTO TWO LEVELS: <u>ZERO</u> AND <u>ONE</u>.



HENCE FOR HARD DECISIONS;



FOR THE RANDOM VARIABLE $y(t)$, THERE ARE IN THIS CASE TWO CONDITIONS PROBABILITY DENSITY FUNCTIONS;

$$P[y|s_0] \quad \text{AND} \quad P[y|s_1]$$

WITH RESPECTIVE MEANS OF a_0 AND a_1

THESE PROPABILITIES ARE CALLED LIKELIHOOD FUNCTIONS. THAT IS, THE LIKELIHOOD OF s_1 IS $P[y|s_1]$; THE PROBABILITY OF GETTING s_1 GIVEN y.

> SINCE THE DEMODULATOR WORKS WITH HARD DECISIONS, THE DECODER OPERATES AS A <u>HARD-DECISION DECODER</u>

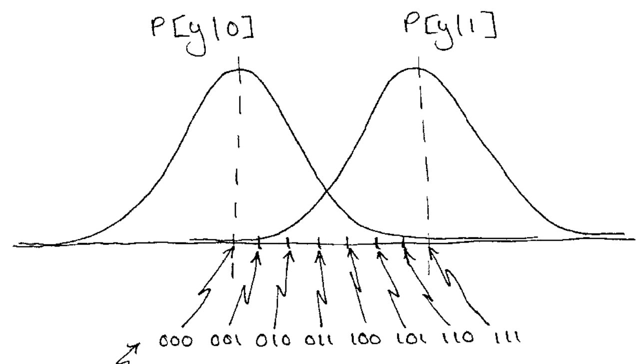
<u>SOFT DECISIONS</u>

WE CAN CONFIGURE OUR DEMODULATOR SUCH THAT IT PROVIDES A QUANTIZED OUTPUT OF MORE THAN TWO LEVELS.

THIS APPROACH GIVES THE DECODER MUCH MORE INFORMATION TO PLAY WITH.

> THIS APPROACH IS CALLED A SOFT DECISION, AND THE DECODING PROCESS IS <u>SOFT-DECISION DECODING</u>

FOR EXAMPLE, IF WE QUANTIZE $y(t)$ INTO 8 LEVELS (3-BITS) WE HAVE



8-LEVEL SOFT DECISION MAKING.

IN EFFECT, WE ARE SENDING A

<u>MEASURE OF CONFIDENCE</u>

ABOUT THE SYMBOL, AS WELL AS THE SYMBOL ITSELF;

SO IF WE GET A "010" WE CAN SAY THAT "WE'RE PRETTY SURE THIS A ZERO", ETC.

OBVIOUSLY AT SOME STAGE WE HAVE TO MAKE A HARD DECISION AT SOME STAGE OR WE WOULD END UP WITH A DECODER OUTPUT LIKE ...

"COULD BE "0110", ... OR IT MIGHT BE "1010", BUT THEN AGAIN IT COULD BE "0111" OR EVEN "1110", BUT THATS NOT SAY THAT IT CAN'T BE "0011" .... "

BUT IF WE LEAVE THE HARD DECISION MAKING TO THE DECODER, WE CAN GENERALLY DO A LITTLE BETTER.

FOR A GAUSSIAN CHANNEL (AWGN) 3-BIT QUANTIZATION GIVES A PERFORMANCE IMPROVEMENT OVER 1-BIT (HARD DECISION) DECODING.

THAT IS;

FOR 8-LEVEL SOFT DECISIONS IF WE HAVE SOME P_E FOR SOME E_b/N_o

THEN FOR 2-LEVEL HARD DECISIONS WE CAN HAVE THE SAME P_E BUT WE REQUIRE $\frac{E_b}{N_o}$ + 2dB.

IF WE TAKE THE IDEA OF SOFT DECISIONS TO THE LIMIT, i.e INFINITE-LEVEL QUANTIZATION IF CAN BE SHOWN THAT THE IMPROVEMENT IS ONLY 2.2dB

FOR THIS REASON SOFT DECISION DECODING IS RARELY USED FOR MORE THAN 3-BITS

WHAT'S THE CATCH?

OBVIOUSLY WE HAVE TO HANDLE MORE INFORMATION IN THE CASE OF SOFT DECISION DECODING;

* FOR BINARY COMMS, HARD DECISIONS WE HAVE 1 BIT-PER-SYMBOL

* FOR SOFT DECISIONS WE MIGHT HAVE 2 OR 3 BITS-PER-SYMBOL; THREE TIMES THE AMOUNT OF DATA.

THE PRICE WE PAY;

* DECODER COMPLEXITY
* DE-MODULATOR COMPLEXITY
* MEMORY
* SPEED.

ALTHOUGH WE CAN USE SOFT DECISION DECODING WITH BLOCK CODES, THE DECODER BECOMES UNWIELDY TO IMPLEMENT.

THE MOST COMMON USE OF SOFT DECISION DECODING IS FOR CONVOLUTION CODES

BINARY SYMMETRIC CHANNEL

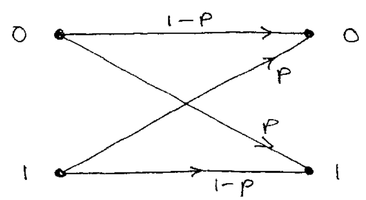
FOR HARD DECISION DECODING, WE OFTEN USE A "BINARY SYMMETRIC CHANNEL" MODEL.

THE BSC CAN BE CHARACTERIZED BY THE FOLLOWING CONDITIONAL PROBABILITIES;

$$P[0|1] = P[1|0] = p$$

$$P[1|1] = P[0|0] = 1-p$$

THAT IS;



WE SAID LAST LECTURE THAT THE MOST LIKELY PATH THROUGH THE TRELLIS IS ONE THAT MAXIMIZES;

$$P[R|C_M]$$

(PROBABILITY OF RECEIVING R GIVEN THAT WE SENT C_M). FOR THE BSC THIS IS EQUIVALENT TO FINDING THE SMALLEST DISTANCE BETWEEN R AND C_M

HAMMING