Transformation of Equational Theories and the Separation Problem of Bounded Arithmetic

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Outline

Motivation

- Polynomial Hierarchy
- Bounded Arithmetic

  Bounded Arithmetic Hierarchy

  Separation Problem
- Consistency Statements

  Gödel's Second Incompleteness Theorem

Aims and Objectives

Background

Theorem (Beckmann (2002))

A pure equational theory whose underlying language is for arithmetic, and inference rules are from equational logic, and whose axioms are based just on recursive axioms; can have its consistency proven in S^1_2.

Theorem (Buss and Ignjatovič (1995))

The equational theory with language L_e, axioms $BASIC_e$, proof system PK and allows inequalities and propositional connectives cannot have its consistency proven in S^1_2.

Task

Find a translation from Buss and Ignjatovič's result into a pure equational setting and prove that S_2^1 cannot prove the consistency of the translated equivalent theory.

Translation is a mapping from boolean formulas to terms with range $\{0, 1\}$.

Translation must be a *good translation* - have the *consistency property* and the *provability property*.

Translation should be formalizable in S_2^1.

Achievements

Results

- Show that S_2^1 cannot prove the consistency of PET - a pure equation theory of Buss and Ignjatovič's result.

- PET - L_p, PI (Symmetry, transitivity, reflexivity, function compatibility), reason in equations, and axioms $BASIC_t$, $BASIC_g$, $BASIC_a$.

- Reminder : Beckmann's result - Any L of arithmetic, rules of equational logic, reasons in equations, and axioms that recursively define the function symbols in the language.

Next Steps

- Replace *function compatibility* with *substitution* in our result.
- Make the set of axioms finite in our result ($BASIC_g$).
- What axioms can be added to Beckmann's result and preserve the provability of the consistency.

References

[1] Arnold Beckmann, *Proving consistency of equational theories in Bounded Arithmetic*. J. Symbolic Logic, 67:279-296. 2002.

[2] Samuel R. Buss and Aleksandar Ignjatovič, *Unprovability of consistency statements in fragments of Bounded Arithmetic*. Ann. Pure Appl. Logic , 74:3, 221-244. 1995.

[3] Stephen Cook *Feasibly constructive proofs and the propositional calculus (preliminary version)*. In Seventh Annual ACM Symposium on Theory of Computing (Albuquerque, N.M., 1975), pages 83-97. Assoc. Comput. Mach., New York, 1975.

Questions?