Transformation of Equational Theories and the Separation Problem of Bounded Arithmetic

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Outline

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Motivation

- Polynomial Hierarchy
- Bounded Arithmetic

  Bounded Arithmetic Hierarchy

  Separation Problem

- Consistency Statements

  Gödel’s Second Incompleteness Theorem
Aims and Objectives
Background

Theorem (Beckmann (2002))
A pure equational theory whose underlying language is for arithmetic, and inference rules are from equational logic, and whose axioms are based just on recursive axioms; can have its consistency proven in $S^1_2$.

Theorem (Buss and Ignjatovič (1995))
The equational theory with language $L_e$, axioms $BASIC_e$, proof system $PK$ and allows inequalities and propositional connectives cannot have its consistency proven in $S^1_2$. 
Task

Find a translation from Buss and Ignjatovič’s result into a pure equational setting and prove that $S^1_2$ cannot prove the consistency of the translated equivalent theory.

Translation is a mapping from boolean formulas to terms with range $\{0, 1\}$.

Translation must be a good translation - have the consistency property and the provability property.

Translation should be formalizable in $S^1_2$. 
Achievements
Results

- Show that $S_2^1$ cannot prove the consistency of $PET$ - a pure equation theory of Buss and Ignjatovič's result.

- $PET$ - $L_p$, $PI$ (Symmetry, transitivity, reflexivity, function compatibility), reason in equations, and axioms $BASIC_t$, $BASIC_g$, $BASIC_a$.

- Reminder: Beckmann's result - Any $L$ of arithmetic, rules of equational logic, reasons in equations, and axioms that recursively define the function symbols in the language.
Next Steps

• Replace *function compatibility* with *substitution* in our result.

• Make the set of axioms finite in our result ($BASIC_g$).

• What axioms can be added to Beckmann’s result and preserve the provability of the consistency.
References


Questions?