Atomic Lambda Calculus and Its Connections with Sharing Graphs

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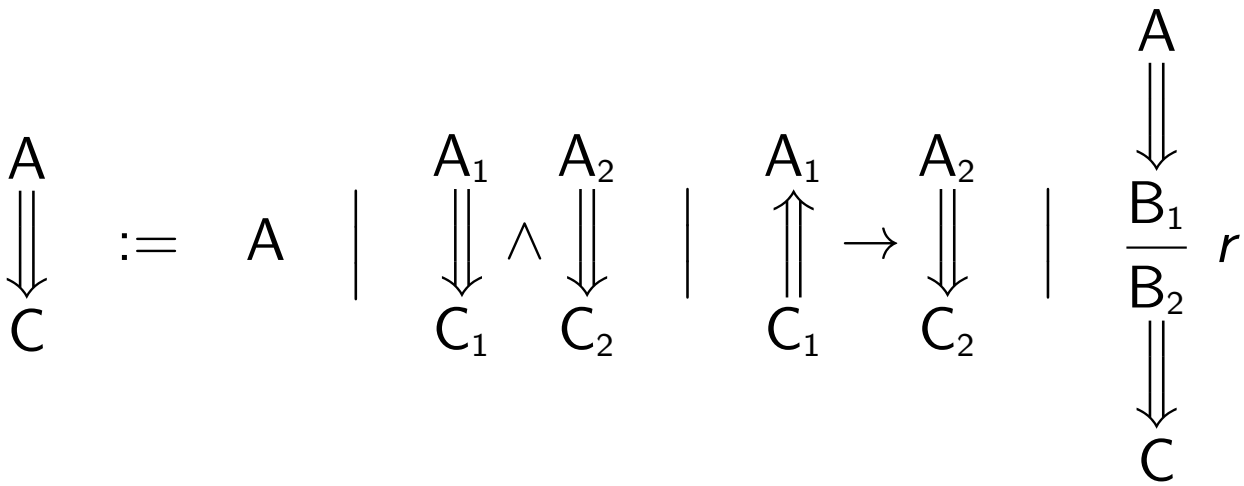
Deep Inference



$$\frac{B}{A \to (A \wedge B)}\ \lambda \qquad \frac{A \wedge (A \to B)}{B}\ @ \qquad \frac{A}{A \wedge \ldots \wedge A}\ \triangle$$

[1] Guglielmi, Gundersen, & Parigot. (2010). A proof calculus which reduces syntactic bureaucracy.

Typing the Lambda Calculus with Explicit Sharing

$$A^x \qquad \dfrac{\Gamma}{A^x \to \boxed{\begin{array}{c} A^x \wedge \Gamma \\ \Downarrow t \\ B \end{array}}}\ \lambda \qquad \dfrac{\boxed{\begin{array}{c}\Gamma \\ \Downarrow u \\ A\end{array}} \wedge \boxed{\begin{array}{c}\Delta \\ \Downarrow t \\ A \to B\end{array}}}{B}\ @$$

$$x \qquad \lambda x.t \qquad (t)u$$

$$\boxed{\Delta \quad \wedge \quad \boxed{\begin{array}{c} \Gamma \\ \Downarrow t \\ A \\ \hline A^{x_1} \wedge \ldots \wedge A^{x_n} \end{array}}\ \triangle} \\ \Downarrow u \\ C$$

$$u[x_1, \ldots, x_n \leftarrow t]$$

The Atomic Lambda Calculus

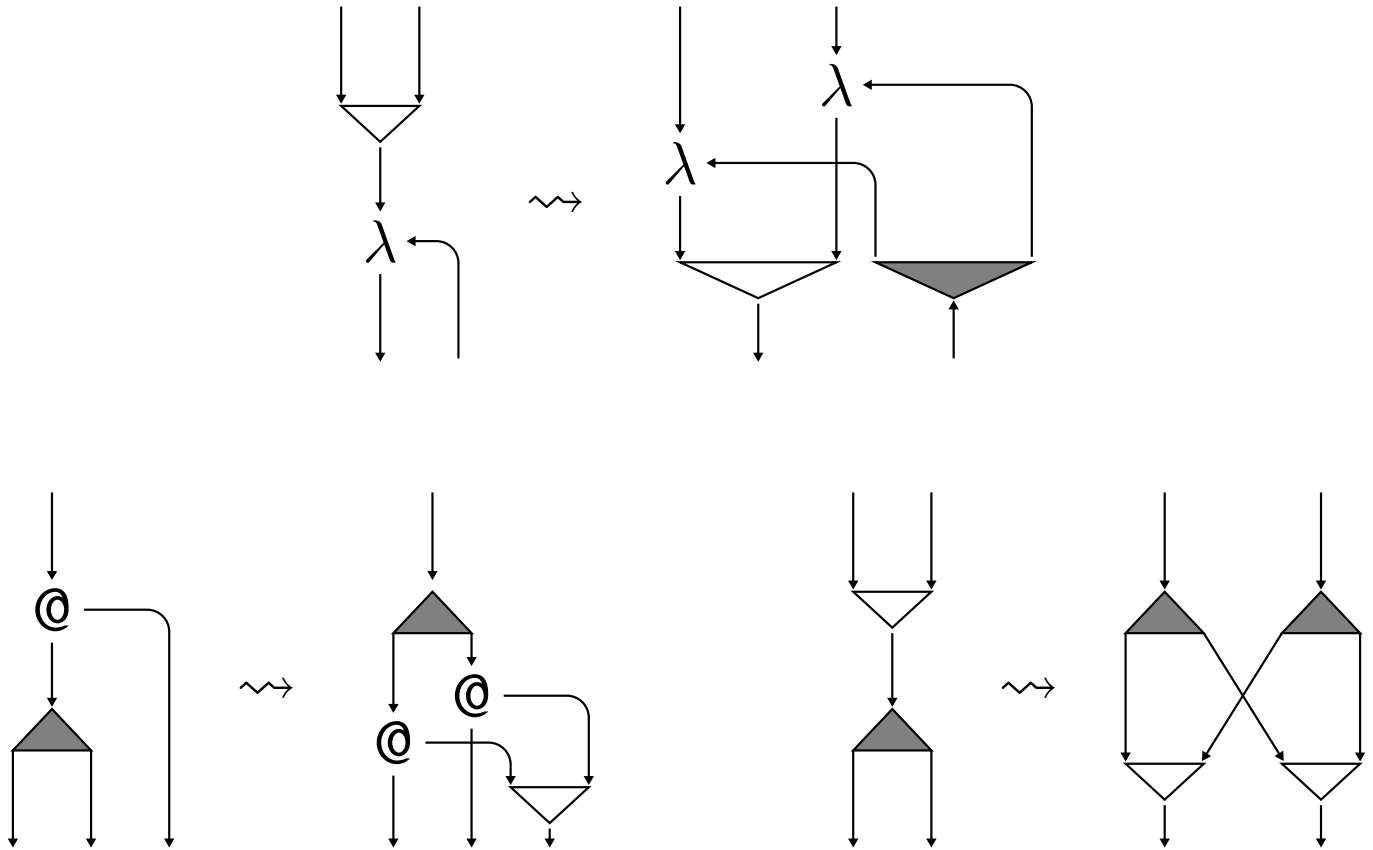
$$\frac{A \to B}{(A \to B) \wedge (A \to B)} \bigtriangleup \rightsquigarrow \frac{\dfrac{A \vee A}{A} \blacktriangledown \quad \to \quad \dfrac{B}{B \wedge B} \bigtriangleup}{(A \to B) \wedge (A \to B)} m$$

Atomic Lambda Calculus implements Fully Lazy Sharing

[2] Gundersen, Heijltjes, & Parigot. (2013). Logic in Computer Science

[4] Wadsworth. Semantics and Pragmatics of the Lambda-Calculus. Diss. University of Oxford, 1971.

Sharing Graphs



Question: Can we obtain optimal reductions with global typing?

[3] Lamping. (1989) ACM.

References

[1] Guglielmi, A., Gundersen, T., & Parigot, M. (2010). A proof calculus which reduces syntactic bureaucracy.
[2] Gundersen, T., Heijltjes, W., & Parigot, M. (2013, June). Atomic lambda calculus: A typed lambda-calculus with explicit sharing. In Proceedings of the 2013 28th Annual ACM/IEEE Symposium on Logic in Computer Science (pp. 311-320). IEEE Computer Society.
[3] Lamping, J. (1989, December). An algorithm for optimal lambda calculus reduction. In Proceedings of the 17th ACM SIGPLAN-SIGACT symposium on Principles of programming languages (pp. 16-30). ACM.
[4] Wadsworth, C. Semantics and Pragmatics of the Lambda-Calculus. Diss. University of Oxford, 1971.