EXAM INFORMATION FOR MA20217: ALGEBRA 2B

1 MAY 2018

Exam format and past papers. The Algebra 2B exam follows the standard format for level 2 units introduced this year, namely *answer all questions from Section A* (worth 40%, testing basic definitions and facts) *and two out of three questions from Section B* (worth 60%, more challenging questions). Although this format is different from previous years, past papers are an excellent guide to the content of the exam. Most past paper questions begin with a few elementary definitions and facts, which would now be Section A material, with the remainder of the question being much like a Section B question in the new format. There are some minor exceptions, listed below, due to small variations in the way the material was taught in a given year.

- 2016/17 All questions are suitable.
- 2015/16 All questions are suitable apart from Q2(d)(ii) (we did not cover products of rings).
- 2014/15 All questions are suitable apart from Q2(c)(iii), and Q3 has too much unseen material based on what was taught this year. Note that Q2(a)(ii) is the theorem that every PID is a UFD (using the definition of a UFD).
- 2013/14 All questions are suitable apart from Q2(c) (products of rings again). The Fundamental Isomorphism Theorem is another name for the First Isomorphism Theorem.
- 2012/13 All questions are suitable. Note that Q2(a)(ii) is the theorem that every PID is a UFD. Also all rings are assumed to be rings with 1.

Long proofs. The aim of the exam is to evaluate your understanding of the material in the unit in a reasonable way, not your ability to learn-by-rote.

Together with past papers, the lecture notes and problem sheet questions marked (W) and (H) provide a good guide to how the material will be examined, but there are some exceptions concerning results with long or off-topic proofs. Note that these exceptions are primarily stating what you will *not* be asked to do in the exam (because it would not be reasonable!).

(1) You will not be asked to reproduce proofs of the following results, either in detail or in outline. However, you may be asked to state or apply them.

Lemma 2.27 and Theorem 2.29	=	The field of fractions of an integral domain
Uniqueness for Theorem 3.21	=	Uniqueness part of 'Every PID is a UFD'
Corollary 3.22	=	Fundamental Theorem of Arithmetic
Lemma 3.24	=	Highest common factors in a UFD
Lemma 3.28 and Corollary 3.29	=	Gauss' lemma
Theorem 3.30	=	Polynomial rings are UFD's
Proposition 4.8	=	Isomorphism of $R[x_1, \dots, x_n]$ with $R[x_1, \dots x_{n-1}][x_n]$
Theorem 4.21	=	Classification of normed algebras
Lemma 5.15	=	Maps on a direct sum of invariant subspaces
Theorem 5.22	=	Jordan normal form - special case
Theorem 5.25	=	Primary decomposition
Lemma 5.30	=	On the generalised eigenspace
Theorem 5.32	=	Jordan decomposition

(2) You will not be asked to reproduce detailed proofs of the following results. However, in addition to being asked to state or apply them, you may be asked to sketch a proof, which means to provide a coherent summary of the main ideas in the proof.

Theorem 2.14 (including 2.11)	=	The First Isomorphism Theorem
Theorem 3.11	=	Euclidean domains are PIDs
Theorem 3.17 (including 3.15)	=	Quotients of PIDs by irreducibles
Proposition 3.20 (including 3.7)	=	Primes and irreducibles in UFDs
Theorem 3.21 (existence only)	=	Any PID is a UFD (existence of factorizations only)
Theorem 4.13 and Theorem 4.15	=	Constructing intermediate fields and field extensions
Theorem 5.7	=	The Cayley–Hamilton Theorem
Proposition 5.19	=	Jordan block matrix representation
Proposition 5.24	=	Primary decomposition into two invariant subspaces
Corollary 5.27	=	Diagonalizability criterion

Sketching proofs of results is something worth doing as a matter of course anyway in all your pure mathematics units, in order to understand the statements of the results, why they are true, and how to use them. If you understand why a result is true in terms of content preceding that result, and can explain it, then that is a good sketch.