Encouraging students to write mathematics properly

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Outline

- The problem.
- My solution(s).
- The way forward.

The problem

Example

Year 1 question: Is $\{1, 2, 3, 4, 5, 6, 7\}$ under multiplication mod 8 a group?

Student answer: not closed. set contains 0.

- 'But you are a lecturer, you know what I meant'.
- 'I've got the correct answer. There it is – see, underlined – at the bottom of the page.'

Does it matter?

Given $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that

$$|a_n - L| < \epsilon \text{ for all } n > N.$$

$$\forall \epsilon > 0 \exists N \in \mathbb{N}(n > N \implies |a_n - L| < \epsilon).$$

$$\exists N \in \mathbb{N} \forall \epsilon > 0(n > N \implies |a_n - L| < \epsilon).$$

Writing mathematics well

It is important to write mathematics correctly.

- Students should be getting credit for showing their intelligence, not hoping that the reader/marker is intelligent enough to work out what is intended.
- Sorting through a jumble of symbols and half-baked, poorly expressed ideas will annoy an examiner or referee. Not a good recipe for obtaining a degree (or at postgraduate level, getting a paper accepted).
- Writing well in any subject is a useful skill to possess. It is highly prized by employers.
- Bonus: Clarifies to you the material you are writing about. (If I can't explain an idea in writing, then I don't understand it.)

Writing mathematics

- Forces thinking.
- Fantastic skill to have.
- Marking is a joy.

Myth

Myth:

Learning mathematics teaches

- critical thinking and
- logical thinking

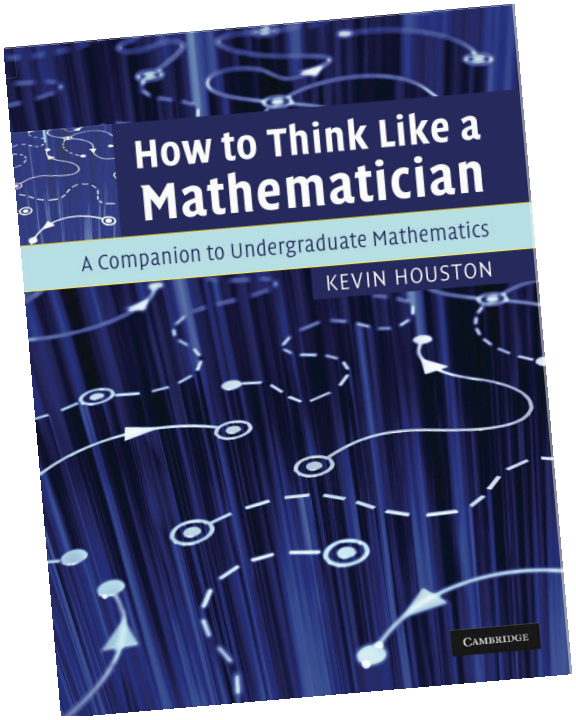
Myth 2

Just tell them to write properly.

‘Tell students to write their work so that someone else can understand it’.

If only it were that simple.

My approach



- Teach thinking.
- Teach writing.

Thinking and writing

To write clearly, students need to think clearly.

To think clearly, students need to write clearly.

Reasons

- A good university education prepares a student for life.
- Writing well is an extremely useful skill.
- Students choose mathematics as they do not like writing essays.
- Low hanging fruit: Small effort gives major boost for student.

Vivaldi

Vivaldi talk. What lecturers provide and what students register.

https://youtu.be/c4cL5HbI_ww?t=9m

Context: Mathematics at the University of Leeds

- 186 intake on BSc/MMath Mathematics.
- Standard offer AAA.
- About 60% have A level FM.
- In 2015/16, we parent 898 UG students.
- In sem 2 of 2015/16 we had a total of 1296 students on our modules.

How to write mathematics

How to write mathematics booklet.

Chapters 3 and 4 of my book.

Copies of samples on my website.

`http://www.kevinhouston.net/httlam.html`

What the students are told

- Write in sentences. This advice has precedence over all others. This can really change the way you present your work.
- Use punctuation. That means a capital letter at the start of sentences and a full stop at the end. Doing this makes your work so much clearer.

What the students are told

- Use grammar correctly.
- Common error: ‘Mathematics is highly symbolic so if I just provide some mathematical symbols I’m doing maths.’ This is wrong. Symbols are merely shorthand for certain concepts; they need to be incorporated into sentences for there to be any meaning.

What the students are told

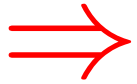
Others:

- Readers are not psychic. Explain what you are doing.
- Explain your assertions
- Use of symbols
- Expressing yourself clearly

My favourite

Curse of the implication symbol.

I can understand why students use the implication symbol:



It makes a proof or the answer to an exercise look mathematical.

I ban its use (during welcome lectures!) until implications have been explained.

Another example

Find, in the form $y = mx + c$, the invariant lines of the transformation with matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Solution

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} mx+c \\ x \end{bmatrix}$$

$$\Rightarrow x = m\left(mx+c\right)+c$$

$$\Rightarrow \left(1-m^2\right)x = \left(m+1\right)c, \quad \text{for all } x.$$

Put $x = 0$, then $\left(m+1\right)c = 0 \quad \Rightarrow m = -1$ or $c = 0$.

Then $\left(1-m^2\right)x = 0$, for all $x \quad \Rightarrow m = \pm 1$.

So, $\quad m = -1 \quad$ or $\quad m = 1, c = 0$

$\Rightarrow y = -x + c \quad$ or $\quad y = x$.

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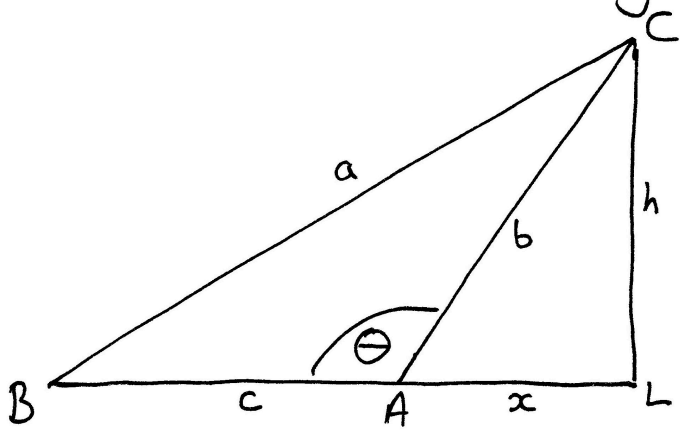
Worked example

Geometry Examples 1



Θ = obtuse angle

$\triangle CBL$

$a^2 = (c+x)^2 + h^2$

$a^2 = c^2 + 2cx + h^2 + x^2$

$\triangle CLA$

$b^2 = h^2 + x^2$

$a^2 = b^2 + c^2 + 2cx$

In $\triangle CLA$ $\quad \frac{x}{b} = \cos(180 - \theta)$

$\Rightarrow x = -b\cos\theta$ Sub into

$a^2 = b^2 + c^2 - 2cb\cos\theta$

Worked example

In the book and booklet students are taken through the errors in the example and shown how to produce a better solution.

Videos of the example are available:
www.youtube.com/user/DrKevinHouston

Videos

Vivaldi - Concepts vs processes.
https://youtu.be/c4cL5HbI_ww?t=10m44s

Robinson - What really annoys me.
https://youtu.be/MvM5zIqASPc?t=4m10s

Weekly exercises

- Weekly exercises in first year (for most modules).
- Groups of 12 (used to be 6).
- 7 marks for content, 3 marks for ‘mathematical presentation’.
- Open University: Good mathematical communication. (About 10% in Level 1 but not all modules.)
- Training for postgrad markers? Training for lecturers?

Weekly exercises

- No tolerance of errors?
- Plenty of feedback. HTTLAM/HTWM page number.
- Too much feedback?
- Do they read it?
- Student resistance.
- Students dispirited.

Other methods

Find the errors from Complex Analysis work.
http://www.kevinhouston.net/pdf/complex-analysis-find-error.pdf

Conclusion

They hate it
at first
but are grateful at the end.

Not very scientific.

Conclusion

- Colleagues (broadly) in favour.
- Can undermine.
- Resistance to ‘skills’.

The way forward

- Low hanging fruit. Big difference. In practice and in research.
- Chapters 3 and 4 are free on my websites.
  - Leeds School of Maths.
  - www.kevinhouston.net
- Contribute examples of good practice. Make Latex exercises available.
- How to measure effect? Anecdotal evidence.

Thanks

Thanks for listening.