Appendix 2 (extension) – Details of Resistance Gene Models

This is an extended version of the "Appendix 2" that appears in the published paper.

We consider (in separate models) two alternative resistance genes; a maternal-effect gene (R_m) and a filial-effect gene (R_f). The latter is discussed in more detail in the paper and referred to there as R. Both resistance genes act in the diploid host to reduce male-killing, and are inherited in an autosomal fashion. The resistance genes differ in their sex-dependent effects (Figure 1). The recursion equations for the two models (with two male-killer strains) are given below.

The maternal-effect resistance gene acts in a dominant fashion in the female host to reduce the transmission of male-killers to her eggs, with the proportion of eggs infected reduced from α to $d\alpha$ (where $0 < d < 1$). We assume that R_m imposes a cost c on females, and that this cost acts multiplicatively so that the fitness of females homozygous for R_m is $(1-c)^2$.

The filial-effect resistance gene (R_f) acts in a dominant fashion in the male host to completely eliminate male-killers from a proportion $1-g$ (where $0 < g < 1$) of infected male embryos, so that the proportion of males killed is $g\alpha$ rather than α. We assume that R_f imposes a cost c on males, and that this cost acts multiplicatively so that the fitness of males homozygous for R_f is $(1-c)^2$.

Mathematical Details of the Models

The genotype frequencies are represented as follows:

males: y, females: x

	No MK	MK_1 (females only)	MK_2 (females only)
No R gene	y_1, x_1	x_4	x_7
Heterozygous for R gene	y_2, x_2	x_5	x_8
Homozygous for R gene	y_3, x_3	x_6	x_9

Invasion of the Maternal-Effect Resistance Gene

Recursion equations for the dynamics of a two male-killer system with the maternal-effect resistance gene were obtained by following the scheme in Figure 1. It is necessary to carry out recursions on individuals because fitness compensation affects broods as a whole.

	y_1	y_2	y_3
x_1	$x_1 + y_1$	$\frac{1}{2}(x_1 + y_1 + y_2 + x_2)$	$y_2 + x_2$
x_2	$\frac{1}{2}(x_1 + y_1 + y_2 + x_2)$	$\frac{1}{4}(x_1 + y_1 + 2y_2 + 2x_2 + y_3 + x_3)$	$\frac{1}{2}(y_2 + x_2 + y_3 + x_3)$
x_3	$y_2 + x_2$	$\frac{1}{2}(y_2 + x_2 + y_3 + x_3)$	$y_3 + x_3$
x_4	$x_1(1-\alpha_1) + y_1(1-\alpha_1) + x_4\alpha_1$	$\frac{1}{2}((1-\alpha_1)(x_1 + y_1 + y_2 + x_2) + \alpha_1(x_4 + x_5))$	$y_2(1-\alpha_1) + x_2(1-\alpha_1) + x_5\alpha_1$
x_5	$\frac{1}{2}((1-d\alpha_1)(x_1 + y_1 + y_2 + x_2) + d\alpha_1(x_4 + x_5))$	$\frac{1}{4}((1-d\alpha_1)(x_1 + y_1 + 2y_2 + y_3 + 2x_2 + x_3) + d\alpha_1(x_4 + 2x_5 + x_6))$	$\frac{1}{2}((1-d\alpha_1)(x_2 + y_2 + y_3 + x_3) + d\alpha_1(x_5 + x_6))$
x_6	$y_2(1-d\alpha_1) + x_2(1-d\alpha_1) + x_5 d\alpha_1$	$\frac{1}{2}((1-d\alpha_1)(x_2 + y_2 + y_3 + x_3) + d\alpha_1(x_5 + x_6))$	$y_3(1-d\alpha_1) + x_3(1-d\alpha_1) + x_6 d\alpha_1$
x_7	$x_1(1-\alpha_2) + y_1(1-\alpha_2) + x_7\alpha_2$	$\frac{1}{2}((1-\alpha_2)(x_1 + y_1 + y_2 + x_2) + \alpha_2(x_7 + x_8))$	$y_2(1-\alpha_2) + x_2(1-\alpha_2) + x_8\alpha_2$
x_8	$\frac{1}{2}((1-\alpha_2)(x_1 + y_1 + y_2 + x_2) + \alpha_2(x_7 + x_8))$	$\frac{1}{4}((1-\alpha_2)(x_1 + y_1 + 2y_2 + 2x_2 + y_3 + x_3) + \alpha_2(x_7 + 2x_8 + x_9))$	$\frac{1}{2}((1-\alpha_2)(x_2 + y_2 + y_3 + x_3) + \alpha_2(x_8 + x_9))$
x_9	$y_2(1-\alpha_2) + x_2(1-\alpha_2) + x_8\alpha_2$	$\frac{1}{2}((1-\alpha_2)(x_2 + y_2 + y_3 + x_3) + \alpha_2(x_8 + x_9))$	$y_3(1-\alpha_2) + x_3(1-\alpha_2) + x_9\alpha_2$

Male death occurs in broods from mothers $x_4 - x_9$. In each case, the surviving brood members receive fitness compensation depending on the mother's genotype. Hence, the number of progeny in each row of the table above must be multiplied by the appropriate fitness compensation term:

Maternal Genotype	Appropriate fitness compensation term	Abbreviation in invasion term
x_4	$1+\frac{\phi\alpha_1}{2-\alpha_1}$	$f4$
x_5, x_6	$1+\frac{\phi d\alpha_1}{2-d\alpha_1}$	$f5$
x_7, x_8, x_9	$1+\frac{\phi\alpha_2}{2-\alpha_2}$	NA

Females in the next generation then suffer the viability costs of carrying the resistance gene and the male-killer, dependent on their own genotype.

Female Genotype	Viability costs
x_1	-
x_2	$(1-c)$
x_3	$(1-c)^2$
x_4	$(1-U_1)$
x_5	$(1-U_1)(1-c)$
x_6	$(1-U_1)(1-c)^2$
x_7	$(1-U_2)$
x_8	$(1-U_2)(1-c)$
x_9	$(1-U_2)(1-c)^2$

The invasion conditions for R_m in the presence of MK_1 only were found by modifier analysis. Linearised recursions were obtained for the three genotypes in which R_f is heterozygous and MK_2 is absent (y_2, x_2, x_5). x_1, y_1 and x_4 were taken to be at their equilibrium frequencies in the absence of R_f. In matrix form, the linearised recursions become:

$$\begin{bmatrix} y_2' \\ x_2' \\ x_5' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} y_2 \\ x_2 \\ x_5 \end{bmatrix} = \lambda \begin{bmatrix} y_2 \\ x_2 \\ x_5 \end{bmatrix}$$

Hence for invasion, the leading eigenvalue, λ of the resultant 3x3 matrix must be greater than one. It therefore follows that:

$$1 < a + bd + e - ae + cg - ceg + bfg + cdh + fh - afh + i - ai - bdi - ei + aei$$

This revealed that for invasion:

$$1 < (-4 + \alpha_1 c^2 d\, f5\, (U_1-1)\, (p^* -1 - p^* f4 + \alpha_1 p^* f4) + 2 p^{*3} (1 - f4 + \alpha_1 f4)\, (1 - f4 + \alpha_1 f4\, U_1)^2 + p^{*2} (1 - f4 + \alpha_1 f4\, U_1)\, (8\, f4 - 8 - 6\, \alpha_1 f4 - \alpha_1 d\, f5 + \alpha_1 f4\, f5 + \alpha_1 d\, f4\, f5 - 2\, \alpha_1^2 d\, f4\, f5 - 2\alpha_1 f4\, U_1 + \alpha_1 d\, f5\, U_1 - \alpha_1 f4\, f5\, U_1 - \alpha_1 d\, f4\, f5\, U_1 + 2\alpha_1^2 d\, f4\, f5\, U_1) + p^* (10 - 10\, f4 + 4\alpha_1 f4 + \alpha_1 d\, f5 - \alpha_1 f4\, f5 - \alpha_1 d\, f4\, f5 + \alpha_1^2 d\, f4\, f5 + 6\, \alpha_1 f4\, U_1 - \alpha_1 d\, f5\, U_1 + \alpha_1 f4\, f5\, U_1 + \alpha_1 d\, f4\, f5\, U_1 - \alpha_1^2 d\, f4\, f5\, U_1^2) + c\, (2 - \alpha_1 d\, f5 + \alpha_1 d\, f5\, U_1 - p^{*2} (1 - f4 + \alpha_1 f4\, U_1)\, (2\, f4 - 2 - 2\alpha_1 f4 - \alpha_1 d\, f5 + \alpha_1 f4\, f5 + \alpha_1 d\, f4\, f5 - 2\alpha_1^2 d\, f4\, f5 + \alpha_1 d\, f5\, U_1 - \alpha_1 f4\, f5\, U_1 - \alpha_1 d\, f4\, f5\, U_1 + 2\alpha_1^2 d\, f4\, f5\, U_1) + p^* (4\, f4 - 4 - 2\alpha_1 f4 + \alpha_1 f4\, f5 - 2\alpha_1 f4 U_1 - \alpha_1 f4\, f5\, U_1 - \alpha_1^2 d\, f4\, f5\, U_1 + \alpha_1^2 d\, f4\, f5\, U_1^2))) \,/\, (4(p^* -1 - p^* f4 + \alpha_1 p^* f4)\, (p^* -1 - p^* f4 + \alpha_1 p^* f4 U_1)^2)$$

The conditions for invasion of R_m when $\phi = 0.5$ and $U_1 = 0.01$ are plotted in Figure 2.

Invasion of The Filial-Effect Resistance Gene

Recursion equations for the dynamics of a two male-killer system with the filial-effect resistance gene:

	y_1	y_2	y_3
x_1	$y_1 + x_1$	$\frac{1}{2}(y_1 + y_2 + x_1 + x_2)$	$y_2 + x_2$
x_2	$\frac{1}{2}(y_1 + y_2 + x_1 + x_2)$	$\frac{1}{4}(1 + y_2 + x_1 + 2x_2 + x_3)$	$\frac{1}{2}(y_2 + y_3 + x_2 + x_3)$
x_3	$y_2 + x_2$	$\frac{1}{2}(y_2 + y_3 + x_2 + x_3)$	$y_3 + x_3$
x_4	$y_1(1-\alpha_1) + x_4\alpha_1 + x_1(1-\alpha_1)$	$\frac{1}{2}((1-\alpha_1)(y_1 + x_1 + x_2) + \alpha_1(x_4 + x_5) + y_2(1-g\alpha_1))$	$y_2(1-g\alpha_1) + x_5\alpha_1 + x_2(1-\alpha_1)$
x_5	$\frac{1}{2}((1-\alpha_1)(y_1 + x_1 + x_2) + \alpha_1(x_4 + x_5) + y_2(1-g\alpha_1))$	$\frac{1}{4}((1-\alpha_1)(y_1 + x_1 + 2x_2 + x_3) + (1-g\alpha_1)(2y_2 + y_3) + \alpha_1(x_4 + 2x_5 + x_6))$	$\frac{1}{2}((1-\alpha_1)(x_2 + x_3) + \alpha_1(x_5 + x_6) + (1-g\alpha_1)(y_2 + y_3))$
x_6	$y_2(1-g\alpha_1) + x_5\alpha_1 + x_2(1-\alpha_1)$	$\frac{1}{2}((1-g\alpha_1)(y_2 + y_3) + (1-\alpha_1)(x_2 + x_3) + \alpha_1(x_5 + x_6))$	$y_3(1-g\alpha_1) + x_6\alpha_1 + x_3(1-\alpha_1)$
x_7	$y_1(1-\alpha_2) + x_7\alpha_2 + x_1(1-\alpha_2)$	$\frac{1}{2}((1-\alpha_2)(y_1 + y_2) + \alpha_2(x_7 + x_8) + (1-\alpha_2)(x_1 + x_2))$	$y_2(1-\alpha_2) + x_8\alpha_2 + x_2(1-\alpha_2)$
x_8	$\frac{1}{2}((1-\alpha_2)(y_1 + y_2 + x_1 + x_2) + \alpha_2(x_7 + x_8))$	$\frac{1}{4}((1-\alpha_2)(1 + y_2 + x_1 + 2x_2 + x_3) + \alpha_2(x_7 + 2x_8 + x_9))$	$\frac{1}{2}((1-\alpha_2)(y_2 + y_3 + x_2 + x_3) + \alpha_2(x_8 + x_9))$
x_9	$y_2(1-\alpha_2) + x_8\alpha_2 + x_2(1-\alpha_2)$	$\frac{1}{2}((1-\alpha_2)(y_2 + y_3 + x_2 + x_3) + \alpha_2(x_8 + x_9))$	$y_3(1-\alpha_2) + x_9\alpha_2 + x_3(1-\alpha_2)$

Again, male death occurs in broods from mothers $x_4 - x_9$, but in this case, the number of males that die is not solely dependent on the mother's genotype. Infected males have a chance of surviving infection if they carry R_f, so the proportion of males dying is also affected by the father's genotype. The fitness compensation received by the brood therefore depends on the combination of the father and mother's genotype.

Parental Genotypes	Appropriate fitness compensation term	Abbreviation in invasion term
$y_1 x_4$	$1+\frac{\phi\alpha_1}{2-\alpha_1}$	*f1*
$y_1 x_6$, $y_2 x_6$, $y_3 x_4$, $y_3 x_5$, $y_3 x_6$	$1+\frac{\phi g\alpha_1}{2-g\alpha_1}$	*NA*
$y_2 x_5$	$1+\frac{(0.75g\alpha_1+0.25\alpha_1)\phi}{2-(0.75g\alpha_1+0.25\alpha_1)}$	*NA*
$y_1 x_5$, $y_2 x_4$,	$1+\frac{(0.5g\alpha_1+0.5\alpha_1)\phi}{2-(0.5g\alpha_1+0.5\alpha_1)}$	*f5*
All matings involving x_7, x_8 and x_9	$1+\frac{\phi\alpha_2}{2-\alpha_2}$	*NA*

Females in the next generation then suffer viability costs dependent on their genotype. In females it is the cost imposed by the male-killer, while males suffer the viability cost of R_f.

Genotype	Viability costs
$x_1\, x_2\, x_3\, y_1$	-
y_2	$(1-c)$
y_3	$(1-c)^2$
$x_4\, x_5\, x_6$	$(1-U_1)$
$x_7\, x_8\, x_9$	$(1-U_2)$

Invasion conditions for R_f were obtained by modifier analysis in the same way as for R_m.

This revealed that for invasion:

$$1<(-4+2\,p^{*3}(1-f5+\alpha_1 gf5)\,(1-f1+\alpha_1 f1\,U_1)^2+p^{*2}(1-f1+\alpha_1 f1\,U_1)\,(4f1-8-2\alpha_1 f1+4f5-2\alpha_1 f5+2\alpha_1 f5\,f1-2\alpha_1^2 f5\,f1-3\alpha_1 f5g-2\alpha_1 f1\,U_1+\alpha_1 f5\,U_1-2\alpha_1 f1\,f5\,U_1+2\alpha_1^2 f1\,f5\,U_1)+p^*(10-8f1+2\alpha_1 f1-2f5+2\alpha_1 f5-2\alpha_1 f1\,f5+\alpha_1^2 f1\,f5+\alpha_1 f5g+6\alpha_1 f1\,U_1-\alpha_1 f5\,U_1+2\alpha_1 f1\,f5\,U_1-\alpha_1^2 f1\,f5\,U_1^2)-c\,(p^*-1-p^*\mathrm{f1}+\alpha_1 p^* f1\,U_1)\,(2-\alpha_1 f5+\alpha_1 f5\,U_1+2p^{*2}(1-f5+\alpha_1 f5g)\,(1-f1+\alpha_1 f1\,U_1)-p^*(4-2f1-2f5+\alpha_1 f5g+2\alpha_1 f1\,U_1+\alpha_1 f5\,U_1)))\,/\,(4\,(p^*-1-p^*\mathrm{f1}+\alpha_1 p^* f1)\,(p^*-1-p^*\mathrm{f1}+\alpha_1 p^* f1\,U_1)^2)$$

The conditions for invasion of R_f when $\phi = 0.5$ and $U_1 = 0.01$ are plotted in Figure 3.