OD-HS EXAMPLES

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Below are some open deduction derivations, along with the od-hs code used to create them. Note that the Haskell code for the propositional logic derivations must be in a different file to the first order logic derivations, since OdProp and OdFOL clash on multiple definitions. The LaTeX packages `virginialake.sty` and `vlralph.sty` must be loaded for the LaTeX outputs to compile.

1. Propositional Logic

```
import OdProp

ex1 = Infn ((Inf (iDapp a' -\/- wDapp a) eq (a' -\/- cDapp a)) -/\-
      (Inf t aiD ((Inf a' acU (a' -/\- wUapp a')) -\/- a)))
      [(eq,((a' -\/- a) -/\- (a' -\/- a))),(sw2, Disj [a',iUapp a,a])]
```

ex1:

$$
= \dfrac{
= \dfrac{\mathsf{ai}\!\downarrow \dfrac{\mathsf{t}}{\bar a \vee a} \vee \mathsf{aw}\!\downarrow \dfrac{\mathsf{f}}{a}}{\bar a \vee \mathsf{ac}\!\downarrow \dfrac{a \vee a}{a}}
\wedge
\mathsf{ai}\!\downarrow \dfrac{\mathsf{t}}{\mathsf{ac}\!\uparrow \dfrac{\bar a}{\bar a \wedge \mathsf{aw}\!\uparrow \dfrac{\bar a}{\mathsf{t}} \vee a}}
}{
\mathsf{2s}\,\dfrac{(\bar a \vee a) \wedge (\bar a \vee a)}{\bar a \vee \mathsf{ai}\!\uparrow \dfrac{a \wedge \bar a}{\mathsf{f}} \vee a}
}
$$

```
d1 = P (PPos "D_1")
d2 = P (PPos "D_2")
e1 = P (PPos "E_1")
e2 = P (PPos "E_2")

redme = Lin "\\color{red}\\me"

ex21  = (Infn (blue (iDapp a -/\- (cc -/\- (a -\/- bb))))
       [(sw2, (purple (a -/\- cc) -\/- Inf (blue (a' -/\- (a -\/- bb)))
       eq (purple ((a -\/- b) -/\- a')))),
       (redme, red
       ((Inf (a -\/- (a -\/- bb)) eq (cDapp a -\/- bb)) -/\- (cc -\/- a')))])
       -/\- ((d1 -\/- d2) -/\- (e1 -/\- e2))

ex22 = (Inf ((a -\/- bb) -/\- (d1 -\/- d2)) sw2
      ((a -\/- d1) -\/- (bb -/\- d2))) -/\-
      (Inf ((Inf (cc -\/- a') eq (a' -\/- cc)) -/\-
      (e1 -\/- e2)) sw2 ((a' -\/- e1) -\/- (cc -/\- e2)))
```

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```
ex23 = (Inf ((a -\/- d1) -/\- (a′ -\/- e1))
      sw (iUapp a -\/- (d1 -\/- e1)) -\/-
      ((bb -/\- d2) -\/- (cc -/\- e2)))

ex2  = Infn ex21 [(eq, ex22),(sw2,ex23)]
```

ex2:

$$
= \frac{
\mathsf{m}\,\dfrac{
\mathsf{2s}\,\dfrac{\mathsf{ai}{\downarrow}\,\dfrac{\mathsf{t}}{a \vee \bar{a}} \wedge (C \wedge (a \vee B))}{(a \wedge C) \vee =\dfrac{\bar{a} \wedge (a \vee B)}{(a \vee b) \wedge \bar{a}}}
}{
=\dfrac{a \vee (a \vee B)}{\mathsf{ac}{\downarrow}\,\dfrac{a \vee a}{a} \vee B} \wedge (C \vee \bar{a})
} \wedge ((D_1 \vee D_2) \wedge (E_1 \wedge E_2))
}{
\mathsf{2s}\,\dfrac{
\mathsf{2s}\,\dfrac{(a \vee B) \wedge (D_1 \vee D_2)}{(a \vee D_1) \vee (B \wedge D_2)} \wedge \mathsf{2s}\,\dfrac{=\dfrac{C \vee \bar{a}}{\bar{a} \vee C} \wedge (E_1 \vee E_2)}{(\bar{a} \vee E_1) \vee (C \wedge E_2)}
}{
\mathsf{s}\,\dfrac{(a \vee D_1) \wedge (\bar{a} \vee E_1)}{\mathsf{ai}{\uparrow}\,\dfrac{a \wedge \bar{a}}{\mathsf{f}} \vee (D_1 \vee E_1)} \vee ((B \wedge D_2) \vee (C \wedge E_2))
}
}
$$

2. First Order Logic

```
import OdFOL

ex31 = fax (iDapp px)
ex32 = (exx px) -\/- Inf (faz (neg pz)) eq (exx $ faz (neg py))
ex3 = exx $ Inf ((Inf (px) eq (fay px)) -\/- fay (neg py)) mtD
   (fay (px -\/- (neg py)))

ex3 = Infn ex31 [(uD,ex32),(moD,ex33)]
```

ex3:

$$
\mathsf{m_1}{\downarrow}\,\frac{
\mathsf{u}{\downarrow}\,\dfrac{\forall x \boxed{\mathsf{ai}{\downarrow}\,\dfrac{\mathsf{t}}{\bar{P}x \vee Px}}}{\exists x \bar{P}x \vee =\dfrac{\forall z P z}{\exists x \forall z P y}}
}{
\exists x \boxed{\mathsf{m_2}{\downarrow}\,\dfrac{=\dfrac{\bar{P}x}{\forall y \bar{P}x} \vee \forall y P y}{\forall y \left(\bar{P}x \vee P y\right)}}
}
$$

```
ex41 = fau $ fav $ Disj [nDapp "y" v (Conj [pu,neg pv]),(neg pu),pv,nDapp "y" u (Conj [neg
ex42 = fau $ (let d = (exy (pu -/\- neg py) -\/- neg pu) in Inf (exw dd) eq d
    -\/- fax (exx px -\/- exy (neg px -/\- py)))
ex43 = Inf (fax (exy (px -/\- neg py)) -\/- neg px) uD (fax (exy (px -/\- neg py)) -\/- ex
ex44 = Infn (exz p) [(eq,p), (uD, (exx px -\/- (fax.exy)(neg px -\/- py)))]
```

```
    where p = fax (px -\/- exy (neg px -/\- py))
ex45 = Disj [(faz.exy)(pz -/\- neg py),exz(neg pz),exz pz,(fax.exy)(neg px -\/- py)]

ex4 = Infn ex41 [(uD,ex42),(uD,ex43 -\/- ex44),(eq,ex45)]
```

ex4:

$$
\mathsf{=}\,\dfrac{\mathsf{u}{\downarrow}\,\dfrac{\mathsf{u}{\downarrow}\,\dfrac{\forall u\forall v\boxed{\mathsf{n}{\downarrow}\dfrac{Pu\wedge\bar{P}v}{\exists y\big(Pu\wedge\bar{P}v\big)}\vee\bar{P}u\vee Pv\vee\mathsf{n}{\downarrow}\dfrac{\bar{P}v\wedge Pu}{\exists y\big(\bar{P}v\wedge Pu\big)}}}{\forall u\boxed{\mathsf{=}\dfrac{\exists wD}{\exists y\big(Pu\wedge\bar{P}y\big)\vee\bar{P}u}\vee\forall x\Big(\exists xPx\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}}}{\mathsf{u}{\downarrow}\dfrac{\forall x\exists y\big(Px\wedge\bar{P}y\big)\vee\bar{P}x}{\forall x\exists y\big(Px\wedge\bar{P}y\big)\vee\exists x\bar{P}x}\vee\mathsf{=}\dfrac{\exists z\forall x\Big(Px\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}{\mathsf{u}{\downarrow}\dfrac{\forall x\Big(Px\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}{\exists xPx\vee\forall x\exists y\big(\bar{P}x\vee Py\big)}}}}{\forall z\exists y\big(Pz\wedge\bar{P}y\big)\vee\exists z\bar{P}z\vee\exists zPz\vee\forall x\exists y\big(\bar{P}x\vee Py\big)}
$$

```
ex51 = Inf ((fax.exy)(neg px -\/- py) -/\- (exx.fay)(px -/\- neg py)) uU
         (exx (Inf ((exy)(neg px -\/- py) -/\- (fay)(px -/\- neg py)) uU
         (exy (Inf ((neg px -\/- py) -/\- (px -/\- neg py)) cU f))))
ex5 = Inf (Conj [ex4,ex3]) sw
               (Disj (ps ++ [ex51]))
               where ps = [(fax.exy)(px -/\- neg py),exx (neg px), exx px]
```

ex5:

$$
sw\,\dfrac{\mathsf{=}\,\dfrac{\mathsf{u}{\downarrow}\,\dfrac{\mathsf{u}{\downarrow}\,\dfrac{\forall u\forall v\boxed{\mathsf{n}{\downarrow}\dfrac{Pu\wedge\bar{P}v}{\exists y\big(Pu\wedge\bar{P}v\big)}\vee\bar{P}u\vee Pv\vee\mathsf{n}{\downarrow}\dfrac{\bar{P}v\wedge Pu}{\exists y\big(\bar{P}v\wedge Pu\big)}}}{\forall u\boxed{\mathsf{=}\dfrac{\exists wD}{\exists y\big(Pu\wedge\bar{P}y\big)\vee\bar{P}u}\vee\forall x\Big(\exists xPx\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}}}{\mathsf{u}{\downarrow}\dfrac{\forall x\exists y\big(Px\wedge\bar{P}y\big)\vee\bar{P}x}{\forall x\exists y\big(Px\wedge\bar{P}y\big)\vee\exists x\bar{P}x}\vee\mathsf{=}\dfrac{\exists z\forall x\Big(Px\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}{\mathsf{u}{\downarrow}\dfrac{\forall x\Big(Px\vee\exists y\big(\bar{P}x\wedge Py\big)\Big)}{\exists xPx\vee\forall x\exists y\big(\bar{P}x\vee Py\big)}}}}{\forall z\exists y\big(Pz\wedge\bar{P}y\big)\vee\exists z\bar{P}z\vee\exists zPz\vee\forall x\exists y\big(\bar{P}x\vee Py\big)}\;\wedge\;\mathsf{m_1}{\downarrow}\dfrac{\mathsf{u}{\downarrow}\dfrac{\forall x\boxed{\mathsf{ai}{\downarrow}\dfrac{\mathsf{t}}{Px\vee\bar{P}x}}}{\exists xPx\vee\mathsf{=}\dfrac{\forall z\bar{P}z}{\exists x\forall z\bar{P}y}}}{\exists x\boxed{\mathsf{m_2}{\downarrow}\dfrac{\mathsf{=}\dfrac{Px}{\forall yPx}\vee\forall y\bar{P}y}{\forall y\big(Px\vee\bar{P}y\big)}}}}{\forall x\exists y\big(Px\wedge\bar{P}y\big)\vee\exists x\bar{P}x\vee\exists xPx\vee\mathsf{u}{\uparrow}\dfrac{\forall x\exists y\big(\bar{P}x\vee Py\big)\wedge\exists x\forall y\big(Px\wedge\bar{P}y\big)}{\exists x\boxed{\mathsf{u}{\uparrow}\dfrac{\exists y\big(\bar{P}x\vee Py\big)\wedge\forall y\big(Px\wedge\bar{P}y\big)}{\exists y\boxed{\mathsf{c}{\uparrow}\dfrac{\big(\bar{P}x\vee Py\big)\wedge\big(Px\wedge\bar{P}y\big)}{\mathsf{f}}}}}}}
$$

$$
\mathsf{r1}{\downarrow}\,\dfrac{
=\,\dfrac{
=\,\dfrac{\mathsf{t}}{\forall y_1 \forall y_2 \boxed{\mathsf{ai}{\downarrow}\,\dfrac{\mathsf{t}}{Py_1 \vee \bar{P}y_1} \vee \left(\mathsf{aw}{\downarrow}\,\dfrac{\mathsf{f}}{\bar{P}c} \vee \mathsf{aw}{\downarrow}\,\dfrac{\mathsf{f}}{Py_2}\right)}}
}{
\forall y_1 \boxed{\mathsf{r1}{\downarrow}\,\dfrac{\forall y_2 \boxed{\left(\mathsf{w}{\downarrow}\,\dfrac{\mathsf{f}}{\exists x \forall y\left(\bar{P}x \vee Py\right)} \vee \left(\bar{P}y_1 \vee Py_2\right)\right) \vee \left(\bar{P}c \vee Py_1\right)}}{\mathsf{h}{\downarrow}\,\dfrac{\mathsf{r1}{\downarrow}\,\dfrac{\forall y_2\left(\exists x \forall y\left(\bar{P}x \vee Py\right) \vee \left(\bar{P}y_1 \vee Py_2\right)\right)}{\exists x \forall y\left(\bar{P}x \vee Py\right) \vee \forall y_2\left(\bar{P}y_1 \vee Py_2\right)}}{\exists x \forall y\left(\bar{P}x \vee Py\right)} \vee \left(\bar{P}c \vee Py_1\right)}}
}
}{
\mathsf{h}{\downarrow}\,\dfrac{\exists x \forall y\left(\bar{P}x \vee Py\right) \vee \forall y_1\left(\bar{P}c \vee Py_1\right)}{\exists x \forall y\left(\bar{P}x \vee Py\right)}
}
$$