Planar lattices do not recover from forest fires

Ioan Manolescu
Joint work with Demeter Kiss and Vladas Sidoravicius

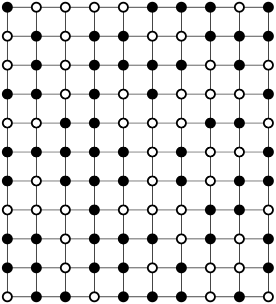
UNIVERSITÉ DE GENÈVE

23 June 2014

What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

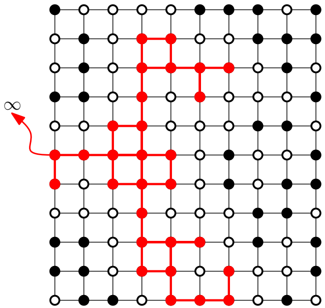
- ω - intensity p (measure $\mathbb{P}_p$).



What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).

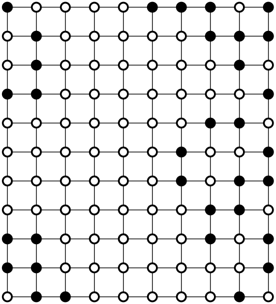


What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).

$\omega \xrightarrow{\text{close } \infty\text{-cluster}} \overline{\omega}$

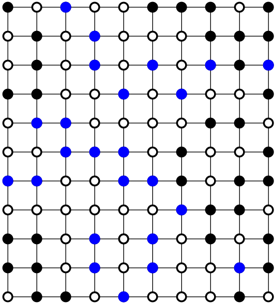


What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).
- σ - intensity δ (small).

$$\omega \xrightarrow{\text{close } \infty\text{-cluster}} \overline{\omega} \xrightarrow{\text{enhancement}} \overline{\omega}^{\delta} = \overline{\omega} \vee \sigma.$$



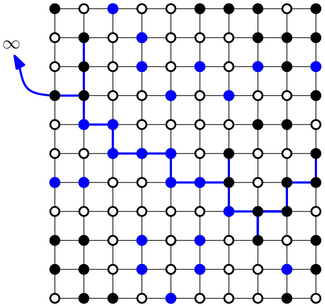
What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).
- σ - intensity δ (small).

$$\omega \xrightarrow{\text{close } \infty\text{-cluster}} \overline{\omega} \xrightarrow{\text{enhancement}} \overline{\omega}^\delta = \overline{\omega} \vee \sigma.$$

$$\delta_c(p) = \sup\{\delta : \mathbb{P}_{p,\delta}(0 \overset{\overline{\omega}^\delta}{\longleftrightarrow} \infty) = 0\}.$$



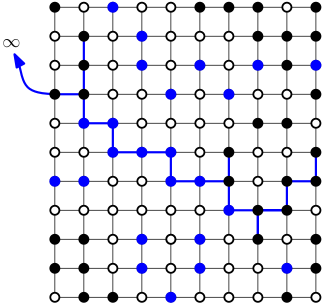
What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).
- σ - intensity δ (small).

$$\omega \xrightarrow{\text{close } \infty\text{-cluster}} \overline{\omega} \quad \xrightarrow{\text{enhancement}} \overline{\omega}^{\delta} = \overline{\omega} \vee \sigma.$$

$$\delta_c(p) = \sup\{\delta : \mathbb{P}_{p,\delta}(0 \overset{\overline{\omega}^{\delta}}{\longleftrightarrow} \infty) = 0\}.$$

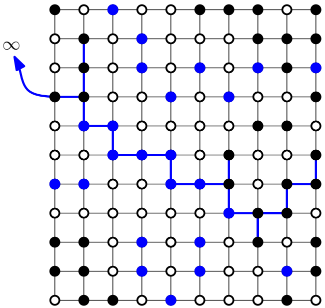


Question: $\delta_c(p) \to 0$ as $p \searrow p_c$?

What is self-destructive percolation?

Let $p, \delta \in [0,1]$.
Two percolation configurations:

- ω - intensity p (measure $\mathbb{P}_p$).
- σ - intensity δ (small).



$$\omega \xrightarrow{\text{close } \infty\text{-cluster}} \overline{\omega} \xrightarrow{\text{enhancement}} \overline{\omega}^{\delta} = \overline{\omega} \vee \sigma.$$

$$\delta_c(p) = \sup\{\delta : \mathbb{P}_{p,\delta}(0 \overset{\overline{\omega}^{\delta}}{\longleftrightarrow} \infty) = 0\}.$$

Theorem [Kiss, M., Sidoravicius] : There exists $\delta > 0$ such that, for all $p > p_c$,

$$\mathbb{P}_{p,\delta}(\text{infinite cluster in } \overline{\omega}^{\delta}) = 0.$$

In particular $\lim_{p \to p_c} \delta_c(p) > 0$

Critical percolation facts

There exists $p_c \in [0,1]$ such that

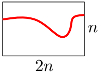
- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

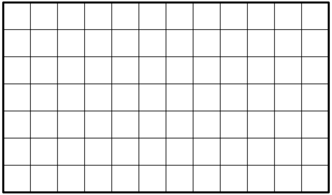
Critical percolation facts

There exists $p_c \in [0,1]$ such that

- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

At p_c..

$\forall n, \ \ \mathbf{P}_{p_c} \Big[$  $\Big] \geq \epsilon$

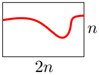


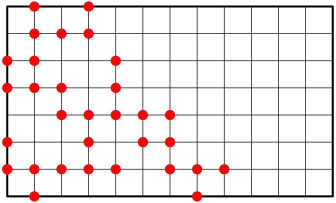
Critical percolation facts

There exists $p_c \in [0,1]$ such that

- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

At p_c..

$\forall n, \quad \mathbf{P}_{p_c}\Big[$  $\Big] \geq \epsilon$

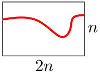


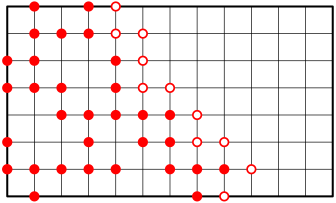
Critical percolation facts

There exists $p_c \in [0,1]$ such that

- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

At p_c..

$\forall n, \ \mathbf{P}_{p_c}$ [] $\geq \epsilon$



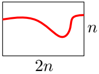
Critical percolation facts

There exists $p_c \in [0,1]$ such that

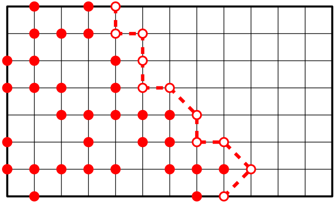
- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

At p_c..

$$\forall n, \quad \mathbf{P}_{p_c}\Bigg[$$



$$\Bigg] \geq \epsilon$$

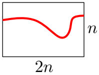


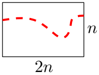
Critical percolation facts

There exists $p_c \in [0,1]$ such that

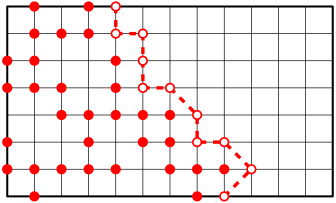
- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

At p_c..

$\forall n,\ \mathbf{P}_{p_c}\Big[$ 

$\Big] \geq \epsilon$ $\quad$ $\mathbf{P}_{p_c}\Big[$ 

$\Big] \geq \epsilon$

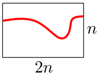


Critical percolation facts

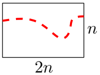
There exists $p_c \in [0,1]$ such that

- $p < p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 0$,
- $p > p_c \Rightarrow \mathbb{P}_p(\text{infinite cluster}) = 1$.

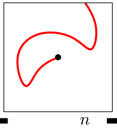
At p_c..

$\forall n,\ \mathbf{P}_{p_c}$ [

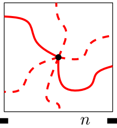
] $\geq \epsilon$

$\mathbf{P}_{p_c}$ [

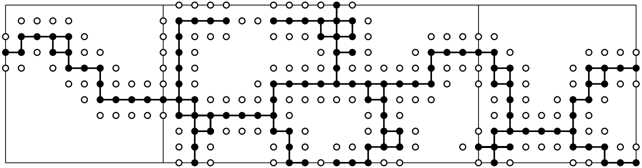
] $\geq \epsilon$

$\mathbf{P}_{p_c}$ [

] $\leq n^{-\alpha_1}$

$\mathbf{P}_{p_c}$ [

] $\leq n^{-(2+\lambda)}$

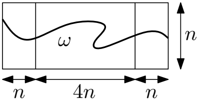


ω containing crossing

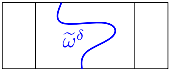
Proposition

For $\delta > 0$ small enough, as $n \to \infty$,

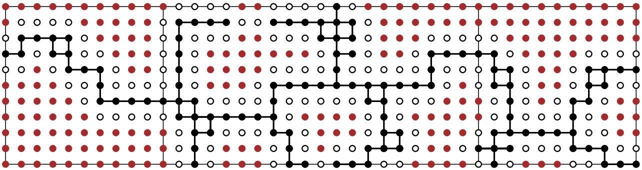
$\mathbb{P}_{p_c,\delta}\Bigg($



and



$\Bigg) \to 0$

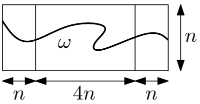


$$\omega \text{ containing crossing} \xrightarrow{\text{delete crossing cluster}} \widetilde{\omega}$$

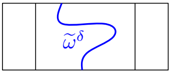
Proposition

For $\delta > 0$ small enough, as $n \to \infty$,

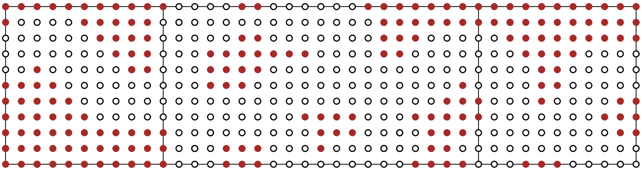
$\mathbb{P}_{p_c,\delta}\Bigg($



and



$\Bigg) \to 0$

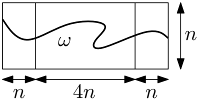


ω containing crossing $\xrightarrow{\text{delete crossing cluster}} \widetilde{\omega}$

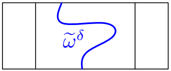
Proposition

For $\delta > 0$ small enough, as $n \to \infty$,

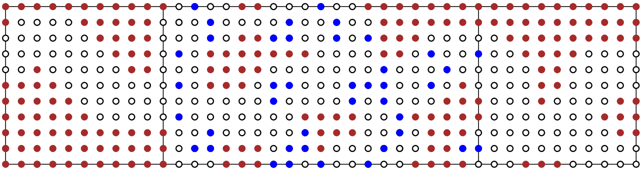
$\mathbb{P}_{p_c,\delta}\Big($



and



$\Big) \to 0$

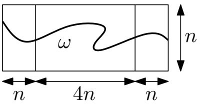


ω containing crossing $\xrightarrow{\text{delete crossing cluster}} \widetilde{\omega} \xrightarrow{\text{enhancement}} \widetilde{\omega}^{\delta}$

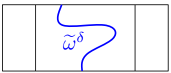
Proposition

For $\delta > 0$ small enough, as $n \to \infty$,

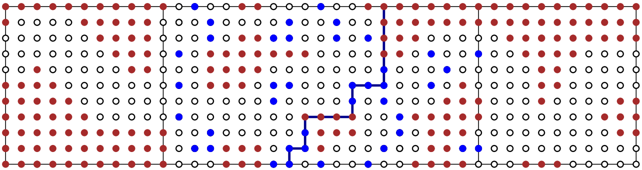
$\mathbb{P}_{p_c,\delta}\Bigg($



and



$\Bigg) \to 0$

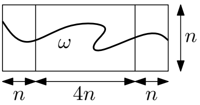


ω containing crossing $\xrightarrow{\text{delete crossing cluster}} \widetilde{\omega} \xrightarrow{\text{enhancement}} \widetilde{\omega}^{\delta}$

Proposition

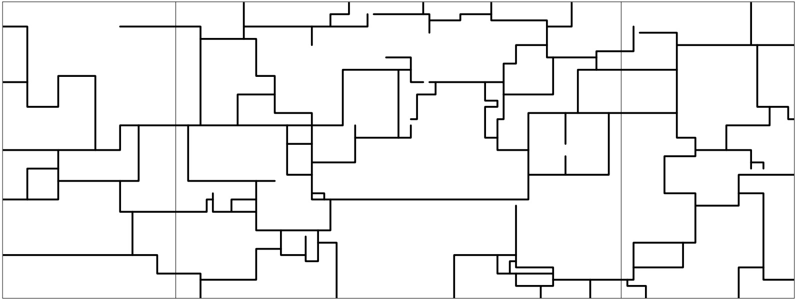
For $\delta > 0$ small enough, as $n \to \infty$,

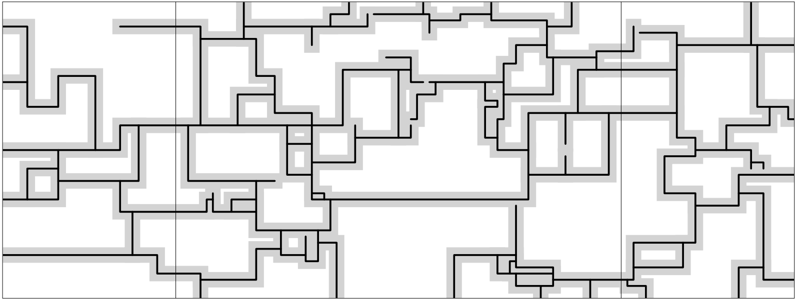
$\mathbb{P}_{p_c,\delta}\Big($

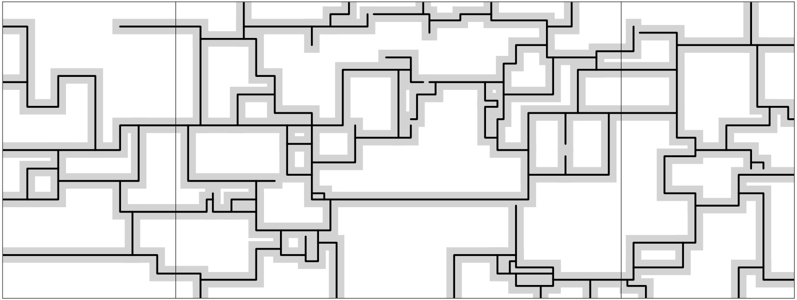


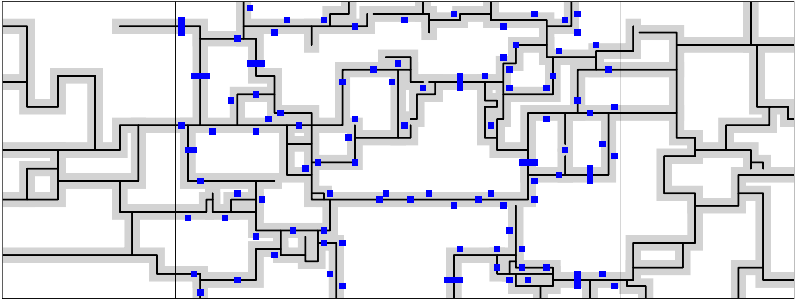
and

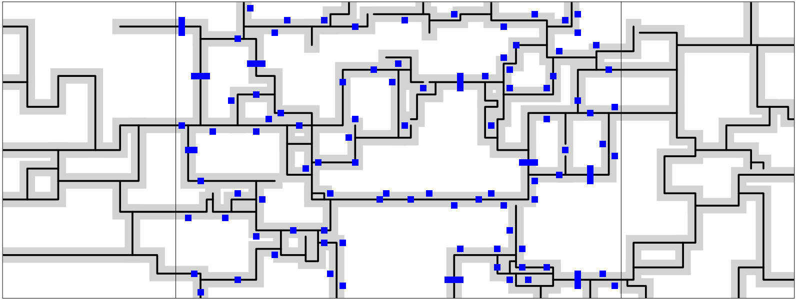
$\Big) \to 0$



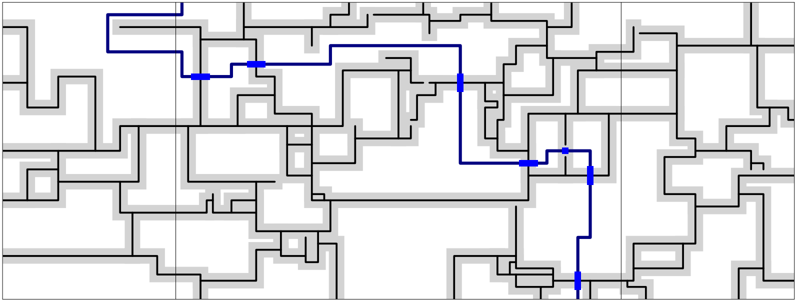








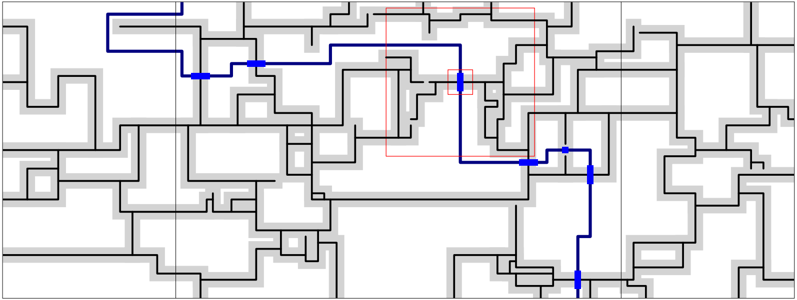
γ - vertical crossing with minimal number of enhanced points.



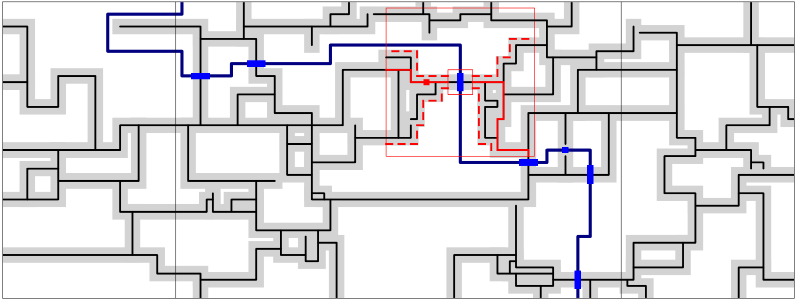
γ - vertical crossing with minimal number of enhanced points.

$\mathcal{X} = \{$enhanced points used by $\gamma\}$. If no crossing $\mathcal{X} = \emptyset$.

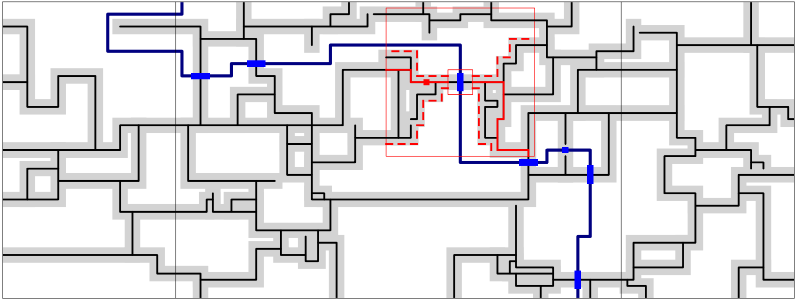
$$\mathbb{P}_{p_c,\delta}(\text{vertical crossing in } \widetilde{\omega}^{\delta}) = \sum_{X \neq \emptyset} \mathbb{P}_{p_c,\delta}(\mathcal{X} = X).$$



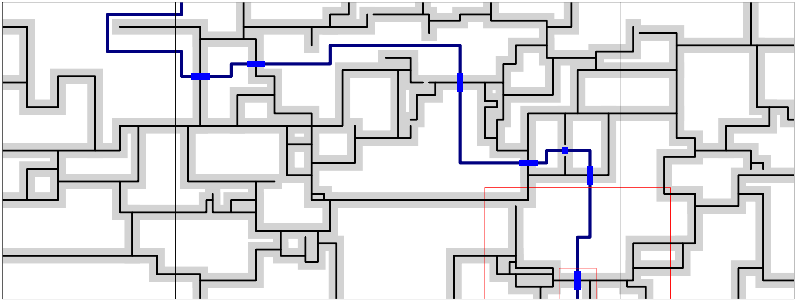
Annulus surrounding passage points but not containing passage points: 6 arms or 4 half-plane arms in ω (possibly with one defect).



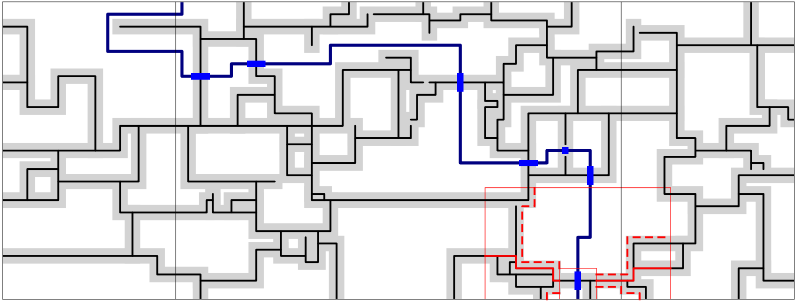
Annulus surrounding passage points but not containing passage points:
6 arms or 4 half-plane arms in ω (possibly with one defect).



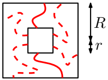
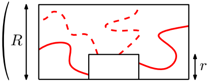
Annulus surrounding passage points but not containing passage points:
6 arms or 4 half-plane arms in ω (possibly with one defect).

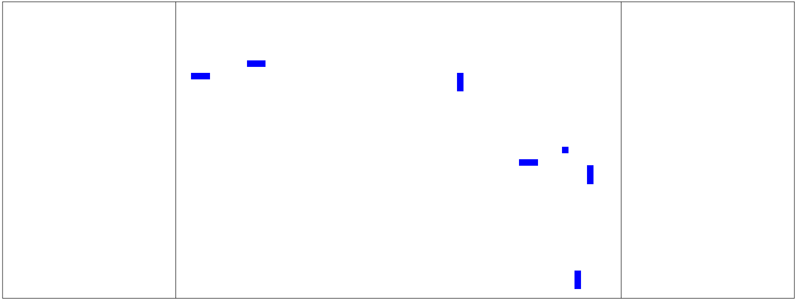


Annulus surrounding passage points but not containing passage points: 6 arms or 4 half-plane arms in ω (possibly with one defect).

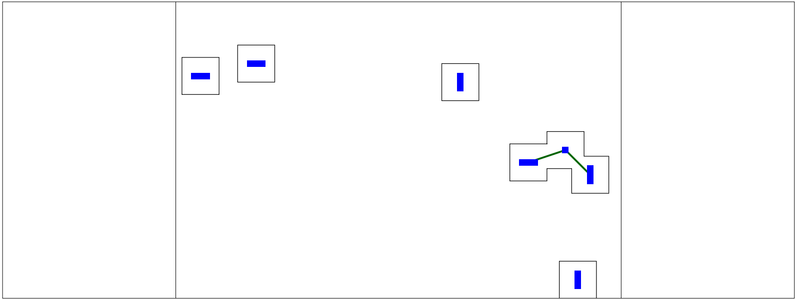


Annulus surrounding passage points but not containing passage points: 6 arms or 4 half-plane arms in ω (possibly with one defect).

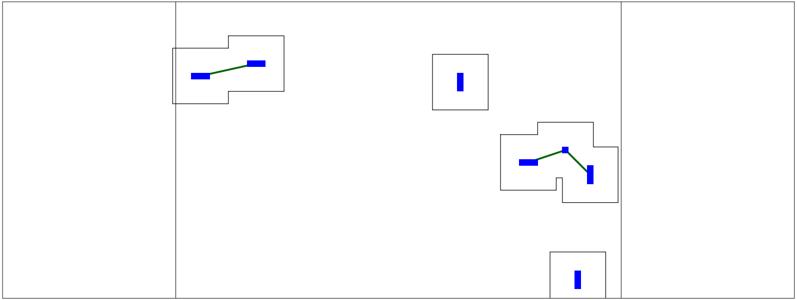
$\mathbb{P}_{p_c}\Big($  $\Big) \leq \left(\frac{r}{R}\right)^{2+\lambda}$ $\mathbb{P}_{p_c}\Big($  $\Big) \leq \left(\frac{r}{R}\right)^{2+\lambda}$



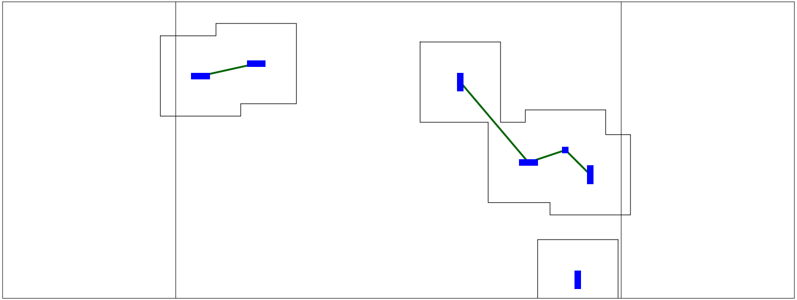
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



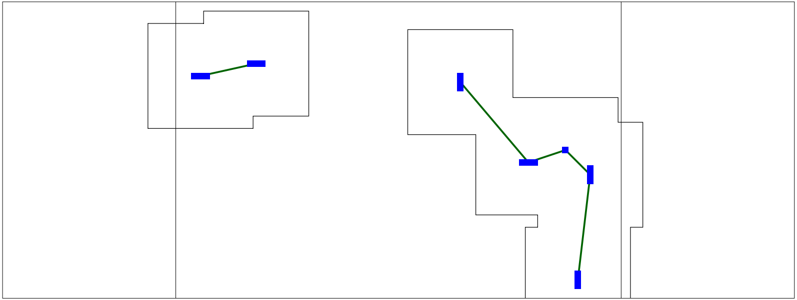
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



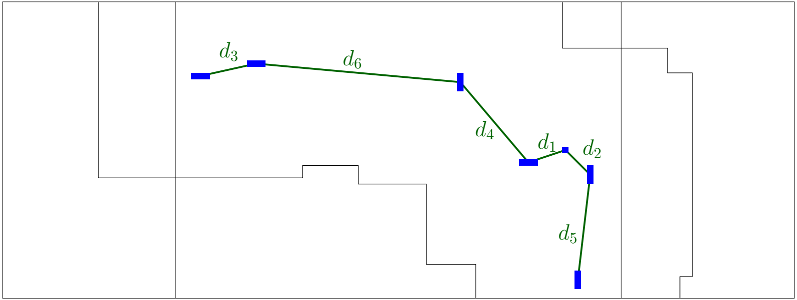
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



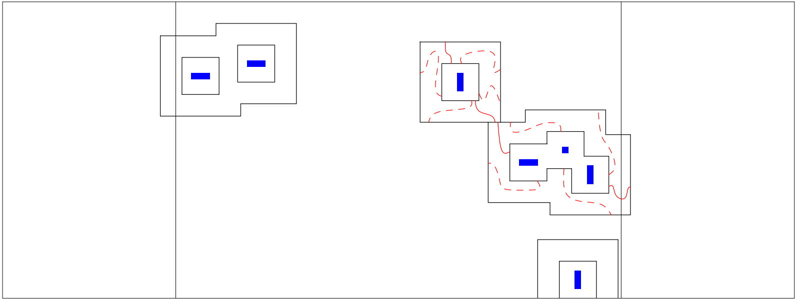
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



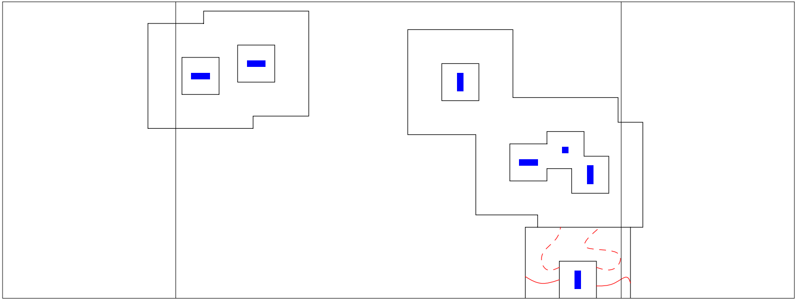
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



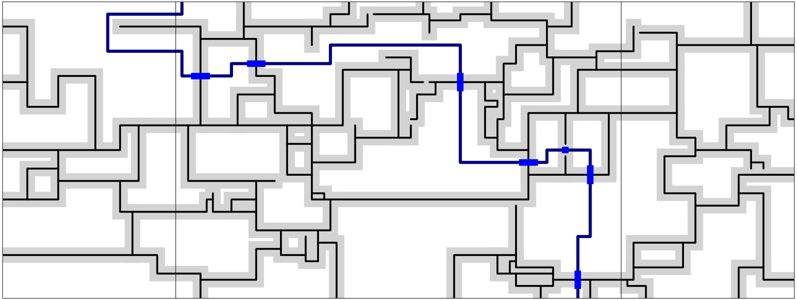
For a set X, what is $\mathbb{P}_{p_c,\delta}(\mathcal{X} = X)$?



$$\mathbb{P}_{p,\delta}(\mathcal{X} = X) \leq c^k n^{-2-\lambda} \prod_j d_j^{-2-\lambda} \times \delta^k,$$

where $d_1, \dots, d_k$ are the merger times of X.

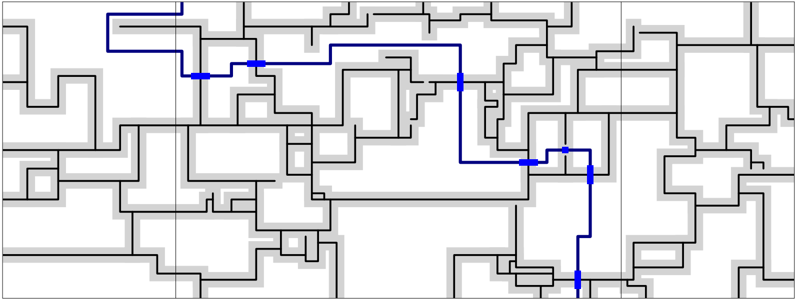
$$\#\{X \text{ with merger times } d_1, \dots, d_k\} \leq C^k n^2 \prod_j d_j.$$



$$\mathbb{P}(\text{vetical crossing in } \widetilde{\omega}^{\delta}) \leq n^{-\lambda} \sum_{\substack{k \geq 1 \\ d_1, \ldots, d_k}} \left(\delta^k c^k \prod_k d_k^{-1-\lambda} \right)$$

$$= n^{-\lambda} \sum_{k \geq 1} \left(\delta c \sum_{d \geq 1} d^{-1-\lambda} \right)^k \to 0,$$

for $\delta > 0$ small.



$$\mathbb{P}(\text{vetical crossing in } \widetilde{\omega}^{\delta}) \le n^{-\lambda} \sum_{\substack{k\ge 1\\ d_1,\dots,d_k}} \left(\delta^k c^k \prod_k d_k^{-1-\lambda} \right)$$

$$= n^{-\lambda} \sum_{k\ge 1} \left(\delta c \sum_{d\ge 1} d^{-1-\lambda} \right)^k \to 0,$$

for $\delta > 0$ small.

Thank you!