Waldram Diagram

Introduction

The Waldram diagram is used to estimate at a single position the direct illuminance from the sky. It is employed in planning when there is a requirement to establish that there will be adequate potential for daylight and in legal cases of Right of Light when the effects of new buildings need to be investigated. Its particular advantage over other methods is its capacity to cope with either complicated window shapes or intricate skylines.

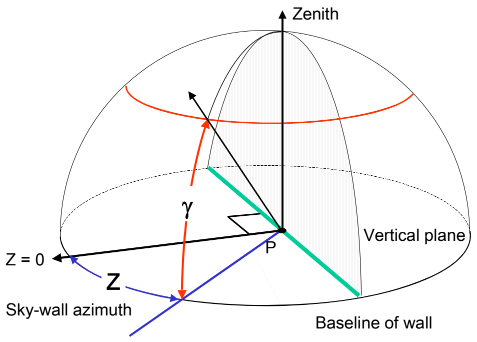

Figure 4.1 - Hemisphere of sky vault

The method is applicable to implementation within a computer package.

Principle of the diagram

A Waldram diagram is a rectangular representation of half of the sky vault shown in Figure 4.1. Figure 4.2 shows a schematic of this representation where the vertical axis corresponds to the altitude γ and the horizontal axis represents the sky-wall azimuth z .

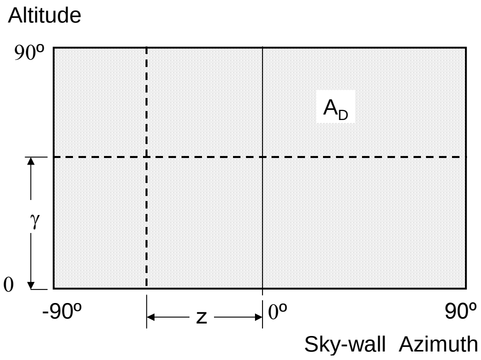

Figure 4.2 – Schematic of Waldram Diagram

The particular feature of the Waldram diagram is that the vertical scale of altitude γ and horizontal scale of z are so contrived as to make equal areas on the diagram represent equal contributions of illuminance from the sky.

A window outline as seen from some position P shown in Figure 4.3, can be plotted on a Waldram diagram as shown in Figure 4.4. Because the diagram is so designed, the sky illuminance at P is proportional to the area within the window outline A_W on the Waldram Diagram;

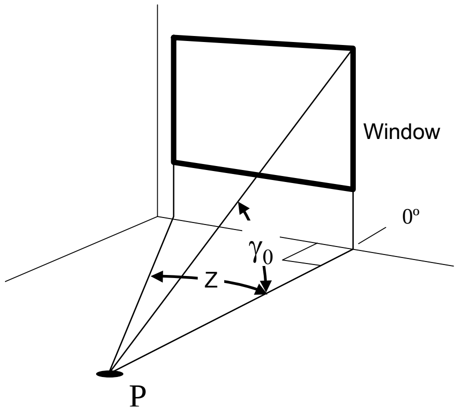

Figure 4.3 – Window illuminating position P

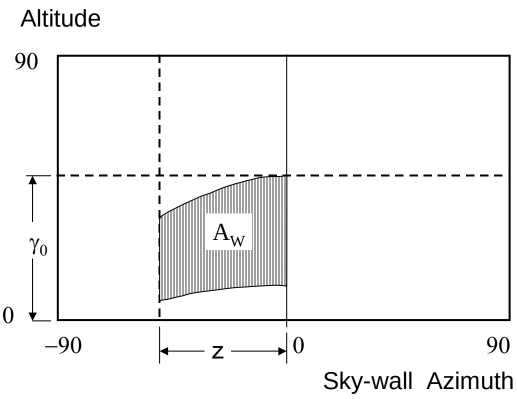

Figure 4.4 – Window plotted on Waldram Diagram

$$E_P \propto A_W \qquad \ldots 1$$

The area of the Waldram diagram represents only half the sky vault and therefore the illuminance from the whole sky will be proportional to twice the area of the diagram A_D,

$$E_{SKY} \propto 2A_D$$

The sky component at P will therefore be given by:

$$SC_P = \frac{E_P}{E_{SKY}} \times 100\% = \frac{A_W}{2A_D} \times 100\% .$$

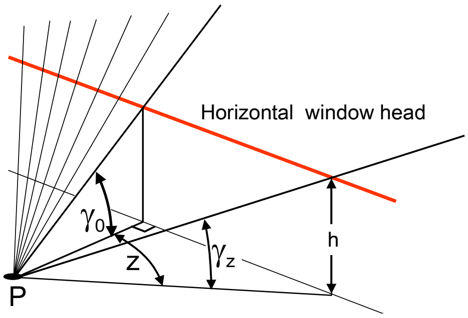

Figure 4.5 – Parallel to window wall

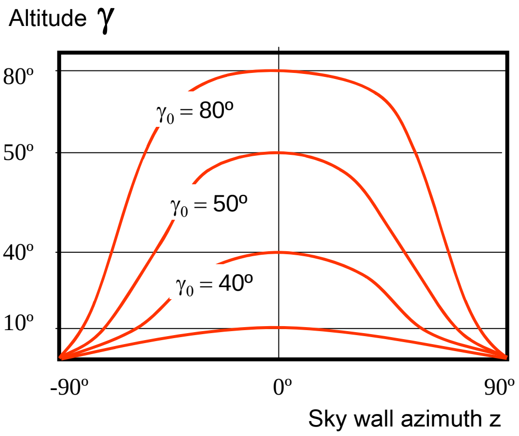

Figure 4.6 – Parallel droop lines on diagram

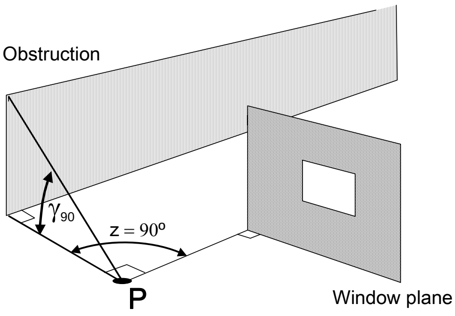

Figure 4.7 – Obstruction perpendicular

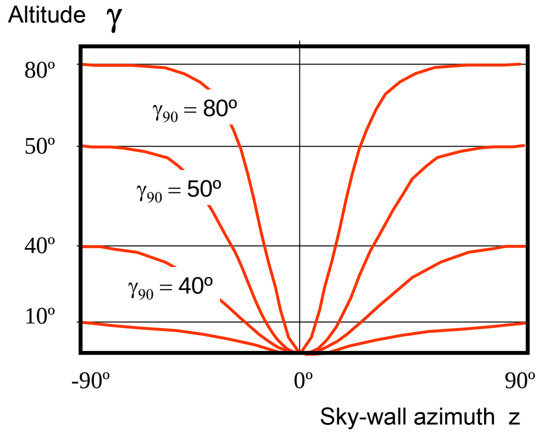

Figure 4.8 – Perpendicular droop lines

Obstructions may also be plotted on the diagram leaving areas of unobstructed sky seen at some position P. Any number of windows may be plotted and their contributions summed.

Also, Waldram diagrams may be constructed for a number of different conditions, such as;

1) Uniform Diffuse or CIE Overcast skies,
2) With or without corrections for transmission through glass,
3) Various scaling factors,
4) Illuminances on either Horizontal or Vertical planes.

Parallel droop lines

The depiction of a window outline on the Waldram diagram is helped by the provision of a series of guide lines that show the line of a window head of constant height. A window of a given height will be at some altitude γ_0 when seen from a position P perpendicular to the window wall. The altitude of the window head will decrease to some angle γ_z at a sky-wall azimuth angle z. When the sky-wall azimuth angle z reaches 90° , then $\gamma_{90} = 0^{\circ}$.

Any horizontal line that lies in the plane swept out from P along the window head will also lie on the appropriate guide line on the Waldram diagram. Lines that are horizontal and parallel to the window wall lie in such planes and thus give rise to the name of the guide lines as 'Parallel droop lines'.

Note the non linearity of the scale of altitude.

Window edges

The vertical sides of a window will have a constant azimuth and they are therefore represented on a Waldram diagram as vertical lines.

Perpendicular droop lines

An obstruction that runs perpendicular to the window wall as shown in Figure 4.7 can be plotted on a Waldram diagram using another series of guidelines. The defining altitude of the obstruction is again the angle measured perpendicular to the line of interest. In the case of an obstruction that is perpendicular to the window wall, the altitude of the obstruction will reduce to 0° as it extends to infinity. Its altitude will therefore be 0° at a sky-wall azimuth of 0°.

In some of the Waldram diagrams there are no labels associated with the perpendicular droop lines. This is because all the various diagrams are produced by a single computer program and there is no simple condition which can be used to position the labels appropriately for all conditions. However, an additional azimuthal line at 45° is provided and this may be used to identify the different droop lines as shown in Figure 4.9. Because the parallel and vertical droop lines of the same altitude intersect at 45°, a perpendicular droop line may be traced to the 45° azimuth line and then the parallel droop line intersecting at that azimuth may be followed to the altitude scale.

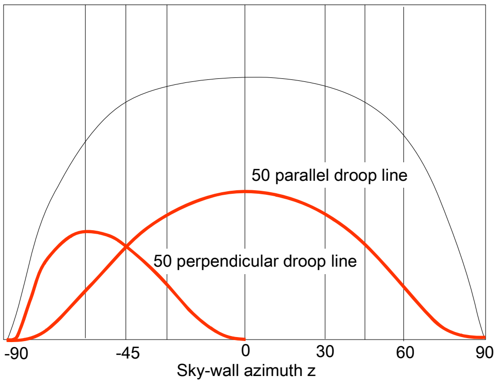

Figure 4.9 – Parallel and perpendicular cross at 45°

Modified diagrams

It is often the case that it is the lower part of the sky that is of particular interest. Introducing a scaling factor to the height of the diagram enables lower altitudes to be considered more conveniently and with increased accuracy. At the edge of each diagram is noted either, the method of directly evaluating the illuminance or, the scaling factor used and the area of diagram representing the whole sky is then given by,

$$A_{SKY} = 2.A_D.Scaling\ Factor.$$

Including a correction for the transmission of light through glass introduces restrictions on how the diagrams can be used. The transmittance of glass depends upon the angle of incidence and as this is not constant for a given azimuth the correction factor calculated for a given azimuth will only be applicable to one orientation of glass. Thus glazing of different orientations must be considered separately.

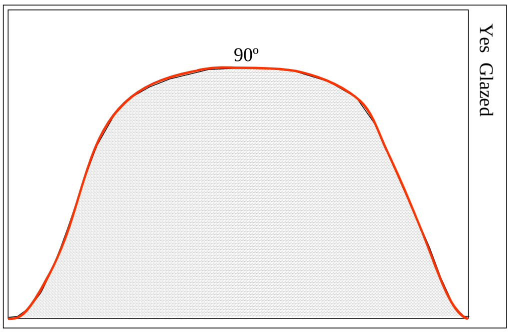

Figure 4.10 – Corrections for glass transmission

The reduced light level that results from using a glazing material is reflected in the contraction of the droop lines in the Waldram diagram as in Figure 4.10. Particularly it can be noted that at an azimuth of 90° the angle of the incident light is 90° and therefore as the light is totally reflected from the glass surface the light transmittance is zero. This results in both the parallel and perpendicular droop lines tending to zero altitude at an azimuth of 90°. The total transmittance of the glazing will be indicated by the ratio of the area within the 90° droop line to the area of the Waldram diagram. This will be found to be about 65% for ordinary single glazing.

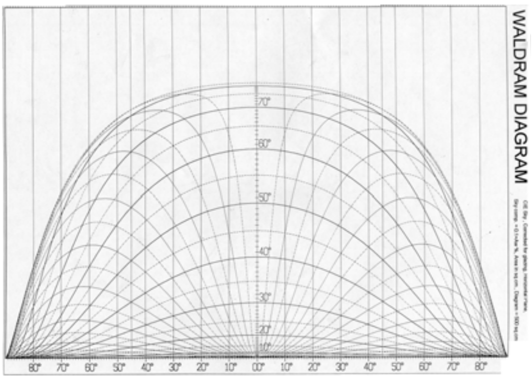

Figure 4.11 – Waldram diagram

Different Waldram diagrams display a number of features. In diagrams for horizontal illuminance the contraction of the droop lines at low angles of altitude is a result of the high angle of incidence of light from the sky on a horizontal plane and the consequent low values of cos*i*. The contraction of the droop lines at higher angles of incidence results from the decreasing solid angles of sky at higher angles of incidence.

In the diagrams for vertical illuminance both the cosine of the angle of incidence and the solid angle subtended at a given altitude increase as the altitude decreases and therefore the droop line scale expands at lower altitudes. Where the sky is a CIE overcast distribution this effect is somewhat countered by the decreasing luminance of the sky at lower altitudes.

An Example

A Waldram Diagram is used here to estimate the horizontal Sky Component and the Externally Reflected Component through glazed windows from a CIE sky.

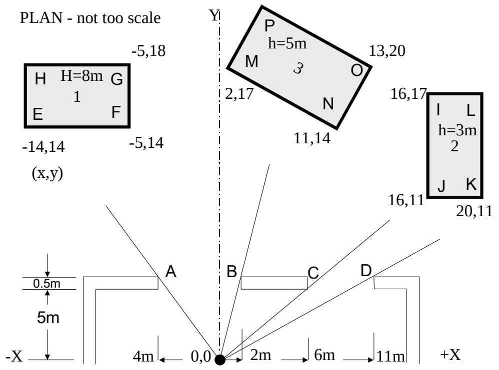

Figure 4.12 – Building plan

Figure 4.12 shows the plan of a room with two windows looking out to three external buildings. The heights of the various buildings are shown on the figure and the window heads are at a height of 6m and cills are at a height of 2m. The x,y coordinates from P, the dot, are shown for the salient features of the buildings.

The horizontal angles to the sides of windows and the vertical corners of buildings may be either measured directly from a scaled plan using a protractor or calculated using trigonometrical relationships. Using letters to identify the various features on plan the distances and angles are as follows in table 1, 2 and 3:

Perpdicular lines	Distances from P		Tangent	Vert. Angle to Perp. lns
	Perp. to Window	Height		
	D_P	h	$\tan\gamma_{90}=\frac{h}{D_P}$	γ_{90}
F - G	-5	8	1·6	58·1
J - I	16	3	0·19	10·6

TABLE 1 - Angles for Perpendicular droop lines

Position	Distances from P		Tangent	Azimuth
	Normal to Window	Parallel to Window		
	D_N	D_P	$\tan z = \frac{D_P}{D_N}$	z
A	5·5	-4	0·73	36
B	5·5	2	0·36	20
C	5·0	6	1·2	50·2
D	5·5	11	2·0	63·4
E	14	-14	1·0	45
F	14	-5	0·36	19·7
G	18	-5	0·28	15·5
H	18	-14	0·78	37·9
I	17	16	0·94	43·3
J	11	16	1·45	55·5
K	11	20	1·82	61·2
L	17	20	1·18	49·6
M	17	2	0·12	6·7
N	14	11	0·79	38·2
O	20	13	0·65	33
P	23	4	0·17	9·9

TABLE 4.2 - Horizontal Angles

Parallel lines	Distances from P		Tangent	Parallel Angle
	Normal to Window	Vertical Height		
	D_N	h	$\tan\gamma_0 = \frac{h}{D_N}$	γ_0
A - B	5·5	6	1·09	47·5
A - B	5·5	2	0·36	20
C - D	5·5	4	0·73	36
E - F	14	8	0·57	29·7
J - R	11	3	0·27	15·3
M	17	5	0·29	16·4
N	14	5	0·36	19·7
O	20	5	0·25	14
P	23	5	0·72	12·3

Table 4.3 – Angles for parallel droop lines

These angles are used to choose the appropriate droop lines that are then plotted onto the Waldram diagram shown in Figure 4.13. It might be noted that the two perpendicular droop lines E_G and K_J could have been drawn without calculation because they continue from corners E and J which are already identified through the parallel droop lines and their azimuths.

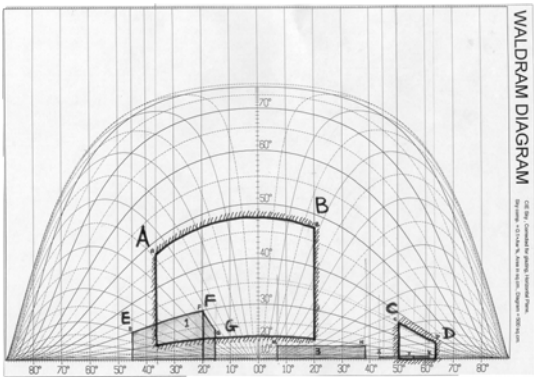

Figure 4.13 – Window and buildings plotted

Building 3 is obliquely positioned relative to the window wall and therefore neither the parallel nor the perpendicular droop lines describe the position of M-N or O-N. Where accuracy is not paramount then a line may be guessed between M and N and O, otherwise, the vertical and horizontal angles of a number of positions along the lines can be calculated and individually plotted on the diagram.

The various areas in cm^2 on the diagram have been measured as:

1) Area of sky seen through window AB = $A_1 = 46$ cm^2
2) Area of sky seen through window CD = $A_2 = 2.5$ cm^2
3) Area of building 1 seen through AB = $A_3 = 3.4$ cm^2
4) Area of building 2 seen through CD = $A_4 = 0.4$ cm^2

Area of diagram representing half the sky = A_D = 500 cm^2

Sky Component at P = $(A_1 + A_2) \div (2 \times A_D) \times 100\%$
= $(46 + 2.5) \div 1000 \times 100\%$
= $48.5 \div 10\%$
= **4.9 %**

Externally reflected component =

$$\frac{\text{Area of Buildings through windows} \times \text{reflectance of buildings} \times 100\%}{\text{Area of diagram representing whole sky} \times 2}$$

$$= \frac{(A_3 + A_4) \times \rho \times 100}{1000 \times 2}\%$$

= $3.8 \times 0.2 \times 0.05$ = **0.04 %**

In this example the externally reflected component may be ignored.