Basic quantities used in lighting

Introduction

In the first year you were introduced to some of the basic units used in lighting and these are shown in Figure 1. Before we embark upon the major part of the course we need to revisit those lighting units originally introduced in the first year and also define some new ones.

A number of units have similar sounding names and it can be easy to confuse them. Practise in their use will help you correctly remember their different meanings.

Basic Lighting units

Φ_v Light flux in **Lumens**
E_v Illuminance in **Lux**
M_v Luminous Exitance in **Apostilbs**

Figure 1 – Units used in 1st. Year

Radiation

All bodies emit electro-magnetic radiation and the amount of radiation emitted by a body is primarily a function of its absolute temperature. In a vacuum, the only mechanism of energy transfer possible is that of radiation exchange. In such an environment, a body will reach a stable temperature when there is no nett exchange of radiant energy between itself and its surroundings i.e. when the body emits energy at the same rate as it absorbs energy from its surroundings.

Note that if the body generates heat internally, then its temperature will rise until it radiates additional energy that is equal to that generated.

The *rate* at which energy is radiated from a hot body is equal to the total power radiated by the body. In these notes this quantity is represented by the symbol Φ_e and has the units of Watts. The suffix 'e' is used to denote the fact that the flow of energy is *electromagnetic radiation.*

The total power radiated by a body is given by the formula,

$$\Phi_e = \varepsilon \sigma A T^4 \quad \text{Watts.}$$

This is restated in Figure 2 together with definitions of all the terms. There is a new quantity referred to and this is ε, the emissivity of the radiating surface. This quantity relates the emitting power of a real surface to that of an ideal theoretical surface. The ideal surface is given many names:

Plankian radiator, Full radiator, Black body.

Theory provides an accurate prediction of the power emitted by such an ideal body and this is given in Figure 3.

$$\Phi_e = \varepsilon \sigma A T^4 \text{ watts}$$

where

Φ_e = power radiated in Watts
ε = emissivity
σ = Stefan-Boltzmann const.
56· 7 x 10^{-12} kW/m^{2}K^4
A = surface area in m^2
T = absolute temperature in Kelvin

Figure 2 – Power radiated from real body

$$\Phi_e^{th} = \sigma A T^4 \text{ watts}$$

where

Φ_e^{th} is the power radiated by a Plankian radiator

Figure 3 – Power radiated from ideal body

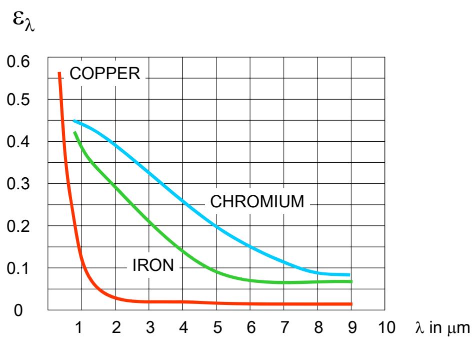

Figure 4 – Spectral emissivity curves
after Lowe 1968

Material	Emissivity
Aluminium - polished	0.095
oxidised	0.20
Copper - polished	0.02
oxidised	0.80
Iron - polished	0.20
oxidised	0.70
Steel - polished	0.07
oxidised	0.80
Stainless steel	0.2 - 0.7
Glass	0.95

After Hottel & Sar

Figure 5 – ε for some materials

The velocity of light in a vacuum is given by,

$$c = \nu\lambda \quad \text{ms}^{-1}$$

and in some material, say glass,

$$c_g = \nu\lambda_g \quad \text{ms}^{-1}$$

Where,
c = velocity in a vacuum in ms^{-1},
c_g = velocity in glass in ms^{-1},
λ = wavelength of light in a vacuum in m,
λ_g = wavelength of light in glass in m,
ν = frequency in Hz.

Figure 6 – Frequency is the invariant

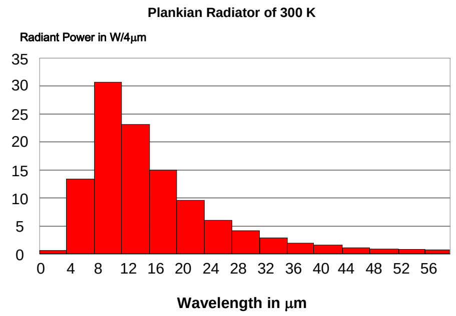

Figure 7 – Spectroradiometric curve

Measurement of the radiation actually emitted from a real surface may be used to estimate the emissivity of that surface at a particular wavelength,

$$\varepsilon_\lambda = \frac{\Phi_{e_\lambda}}{\Phi^{th}_{e_\lambda}} .$$

Figure 4 shows for three metals how the emissivity changes with wavelength. An average or representative value may be chosen from such curves and values from a number of materials are shown in the Figure 5.

When considering the thermal effects of radiation it is often assumed that the spectral composition of the radiation is of minor importance because the emissivity of many surfaces is fairly constant over a wide range of wavelengths. However, the spectral composition of radiation can be important when considering energy transfer within buildings. Particularly because glass has an emissivity in the long wave infra red region that is significantly different to its emissivity at shorter wavelengths.

Also, the distribution of power radiated at different wavelengths is of major importance in lighting because different wavelengths evoke different responses in the receptors in the eye. The power radiated at different wavelengths may be plotted on a diagram known as the *spectroradiometric curve.*

Spectroradiometric curve

In building physics it is common practice to describe a radiation by its wavelength even though a reasonable argument may be put forward to use frequency. This is because frequency is the parameter that remains constant as the radiation passes through different materials: wavelength varies in proportion to the velocity of light in the material. This is summarised in Figure 6.

However, even though frequency is the invariant property of radiation, these notes will continue with the convention of describing radiation in terms of its wavelength.

The power emitted by a radiation source must be measured over a finite range of wavelengths. This bandwidth over which the power is measured may vary from 1nm to 50nm. Lamp manufacturer's will use 1nm wavebands in their literature, but for ease of working these notes will use wavebands of either 25nm or 50nm. Figure 7 shows the spectroradiaometric curve of a source at 300 K where Φ_{e_λ}, the power emitted at different wavelengths is shown in bandwidths of 4μm.

Radiant Power

Φ_e , the total power radiated by this source is found by summing the power emitted over the whole spectrum, i.e.

$$\Phi_e = \sum_{\lambda=0}^{\lambda=\infty} \Phi_{e_\lambda} \Delta\lambda \quad \text{watts} \qquad 1$$

Where;

Φ_{e_λ} = power radiated per $\Delta\lambda$ at wavelength λ in μm ,

and $\Delta\lambda$ = bandwidths over which the power is measured.

Note - in the summation equation 1, the $\Delta\lambda$ cancels with the $\Delta\lambda$ in the definition of Φ_{e_λ} .

Irradiance

This term is used to describe the concentration of radiation incident upon a surface and is measured in watts/m^2. Note that in some books the term *Intensity* is also used to describe this quantity.

$$E_e = \lim_{\delta A \to 0} \frac{\delta \Phi_e}{\delta A} \ \text{W/m}^2 \qquad \ldots 2$$

$$E_e = \frac{\Phi}{\text{A}} \ \text{W/m}^2$$

Radiant Exitance

This term describes the concentration of radiation leaving a surface and is also measured in watts/m^2.

$$M_e = \lim_{\delta A \to 0} \frac{\delta \Phi_e}{\delta A} \ \text{W/m}^2 \qquad \ldots 3$$

$$M_e = \frac{\Phi}{A} \ \text{W/m}^2$$

Note the use of the subscript 'e' to denote that both of the quantities described are electromagnetic radiation.

Spectral Radiant Exitance

The spectral exitance of a Plankian radiator may be evaluated using the equation shown in Figure 8. The spectroradiometric curve shown in Figure 9 was drawn using this equation.

$$M_{e_\lambda}^{Th} = \frac{\lambda^{-5} 3.74 \times 10^{-19}}{e^{\frac{0.01439}{\lambda T}} - 1} \qquad kW/m^2 \mu m$$

$M_{e_\lambda}^{Th}$ Theoretical Spectral Radiant exitance in kW/m^{2}μm

λ Wavelength in μm

T Absolute temperature in Kelvin

Figure 8 - Spectral Exitance of a Black Body

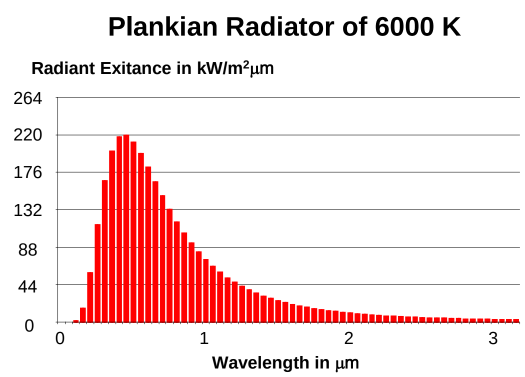

Figure 9 – Spectral exitance of a Black Body

Photometry

Photometry is the science of light measurement and it is one of the foundations upon which lighting engineering is built. It is based upon a number of observations about the way humans respond to light.

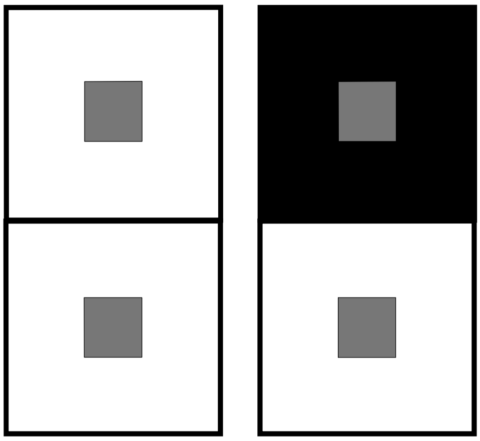

Figure 10 – Brightness is subjective

i) differently coloured surfaces may appear to be equally bright and such surfaces would be said to have equal 'luminosity'.

*We need to distinguish between the terms **luminosity** and **brightness**. A light source that is more luminous than another has a greater **luminosity**, i.e. it emits more light and thus emits a greater stimulus of vision. This is a quantity that can be measured using photocells. However, the term **brightness** is used to describe how great is the sensation that light evokes and it is a subjective quantity that cannot be measured using a simple physical measuring device.*

Two surfaces that appear to be equally bright under identical conditions would have equal luminosities. Therefore the two mid-grey squares on the left of Figure 10 appear to be of equal brightness because they are both surrounded by a white field. However, the two equally luminous squares on the right differ in brightness because the upper square is seen within a dark field.

The important result of (i) is that a quality of luminosity can be identified that is independent of the colour of the light source.

ii) two surfaces seen as being the same brightness by one person will be seen to be the same brightness by most people.

The important result from this observation is that luminosity is a quality that is very nearly the same for most people. It is true that people will see colours and surfaces of equal luminosity slightly differently, but the close similarities are much more significant and important than the slight differences.

This result means that a single measure of luminosity may be used that will be applicable to the general population.

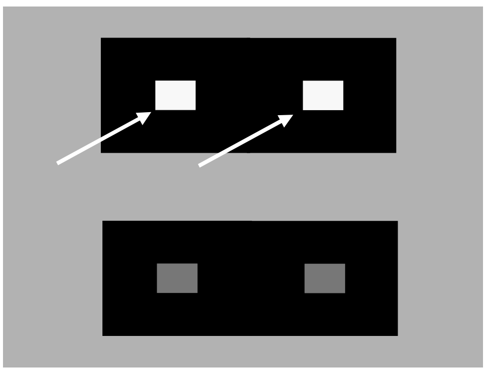

Figure 11 – Luminosity is consistent over a range

iii) if two surfaces have the same luminosity and an equal amount of light is added to each, then the equality of luminosity will not be disturbed, as indicated in Figure 11.

This observation implies that the mechanism that gives rise to luminosity stays fairly constant over a wide range of light levels.

iv) if two coloured light sources are combined with each other the result will be a third coloured light source. If each of the original two colours are separately matched in ***luminosity*** with a fourth and a fifth differently coloured light source, then the ***luminosity*** of the combined fourth and fifth sources will match the ***luminosity*** of the third coloured light source. Figure 12 attempts to show this situation.

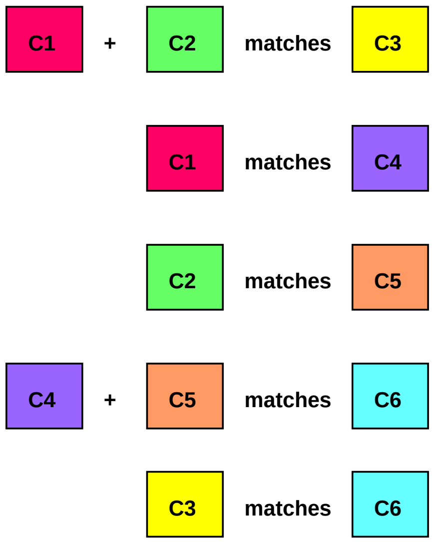

Figure 12 – Additivity of luminosity

This is a most important result. It demonstrates that the property termed luminosity is additive. Thus the luminosity of a surface can be found by summing the effects of a number of component parts. ***This means the overall visual effect of a spectrum may be found by summing the effects of the individual wavelengths of light.***

The origins of photometry

Photometry was originally based upon visually matching surfaces that have identical luminosities. And so in Figure 13, the photometer is positioned so that the illuminances on the two screens in the photometer are the same. This is found by positioning the photometer so that the two illuminated screens on the photometer have the same brightness. Moving the photometer in either direction on the Photometric bench shown in Figure 14, will destroy the equality of brightness. It will be appreciated that if one of the light sources is a 'Standard Light Source' then the relative distances d_1 and d_2 can be used to estimate the luminous power of the other source.

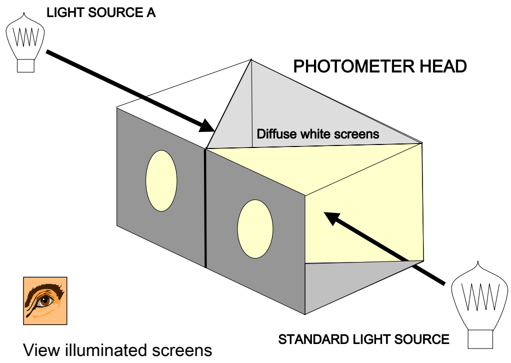

Figure 13 – A photometer

The original standard light source was indeed a candle. Even in the early part of the 20^{th} century the British Standard Candle was;
' a spermaceti candle, $7/8^{th}$ inch diameter, weighting $1/6^{th}$ pound and burning at the rate of 120 grains per hour '.

Such a source varied with the thickness and length of the wick and two other standard light sources were frequently used. The Vernon Harcourt pentane gas lamp and the Hefner lamp burning amyl acetate.

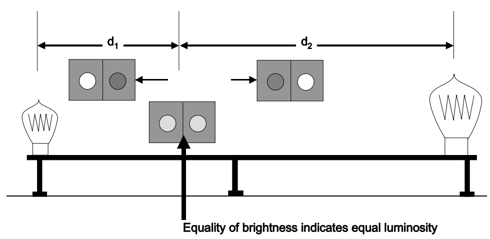

Figure 14 – Photometric bench

A new standard candle was introduced in 1948 and the unit of luminous intensity derived from it was given the name Candela and defined as:
' – one sixtieth of the luminous intensity of 1sq.cm of the surface of a black body at the temperature of solidifying platinum, radiated perpendicular to that surface. '

In 1979, the Candela was alternatively defined as:

' a source of monochromatic radiation of frequency 540×10^{12} Hz and whose radiant intensity is 1/683 watt per steradian. '

Psychophysics

In order to establish the output from a light source, the photometer is positioned so that the unknown light source produces the same ‘ brightness’ on one screen as that produced on the adjacent screen by the ‘Standard Source’. Thus, an equality of brightness is used as the condition of equal luminosity. However, when one surface is more luminous than another it will appear to be brighter in the photometer. Although the source might be said to be brighter, no estimate is made about how much brighter it is. Indeed the brightness of a source or surface can depend upon many factors other than the luminosity of the surface. The effect of the immediate surround has already been mentioned, but additionally there will be effects from adaption and veiling glare. The branch of science that studies the relation between luminosity and the sensation of brightness is *psychophysics.* As designers we are interested in the relative brightness of surfaces and will therefore need to become a little acquainted with the science of *psychophysics*. But this subject will be left for later course.

An alternative way of quantifying light

A measurement of light has been described that relies upon someone visually matching the effect of one light source with the effect produced by a standard source of light. This requires the skill of visually matching two patches of light, and also the availability of a standard source of light.

Another approach to light measurement is possible.

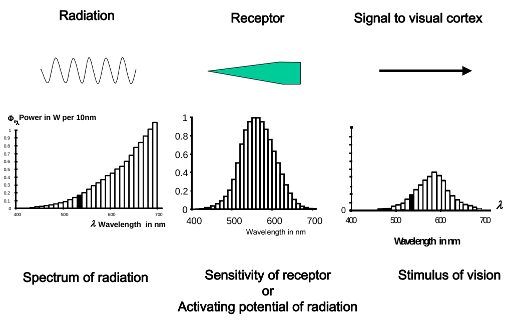

Figure 15 – Measurement of luminous flux

Rather than relying upon visually matching two surfaces, light can be measured directly using a physical instrument. This is done by measuring the power of radiation incident upon a surface and then correcting the output to take into account the radiation’s ability to stimulate the sense of sight.

The first step is the measurement of the incident radiant flux at different wavelengths; the second step is to determine the visual effect of different wavelengths of light. The radiant flux at each wavelength is then corrected for its effectiveness at evoking the sense of sight, and then the effects at individual wavelengths are summed in order to get the total visual effect of the radiation. This process is shown schematically in Figure 15.

Visual Stimulus = Σ (power at λ) x (visual efficiency at λ)

where λ is the wavelength of the radiation.

Standard observers

It should be recognised that there are differences between individuals' responses to different wavelengths of light quite apart from the effects of colour blindness and other visual deficiencies. In order to provide a measure of the effects of wavelength on a 'Standard Observer', the response of a number of individuals has been determined. The average response from these individuals has then been taken as a measure of the effects of different wavelengths of light upon a 'Standard Observer. There will of course be differences between the 'Standard Observer' and the actual response of an individual, but these are likely to be quite small. The slight differences between individuals and the 'Standard Observer' do not invalidate the whole system, but the fact that they exist should be borne in mind when considering lighting measurements.

Photopic, Scotopic and Colour Vision

Also, it should be appreciated that there are different types of receptor within the eye. The two major categories of visual receptor are:

i) Rods that mediate low light vision (Scotopic)
ii) Cones that mediate vision at high levels (Photopic)

The responses of these two receptors are shown in the Figure 16. It might be of interest to note that the scotopic receptors are shifted to shorter wavelengths. Thus the scotopic receptors responsible for night vision are not sensitive to the red end of the spectrum. Hence the use of red light in conning towers etc., so that night time vision is not impaired.

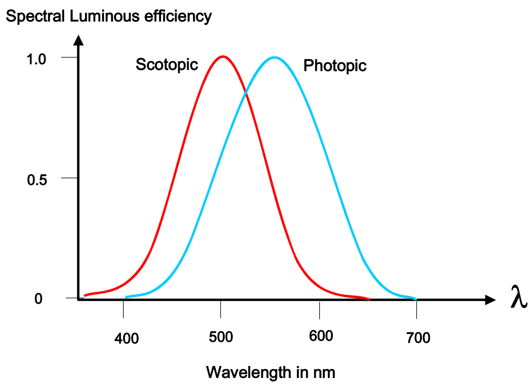

Figure 16 – Sensitivity of light receptors

We will be concentrating on situations where there are high levels and therefore will be considering only Photopic vision.

The cones that mediate photopic vision may be classified into three types,

i) Short wavelength sensitivity (Blue)
ii) Medium wavelength sensitivity (Green)
iii) Long wavelength sensitivity (Red)

The sensitivities of these receptors are shown in a Figure 17. Our sensation of colour results from the relative degree to which these three different types of receptor are stimulated.

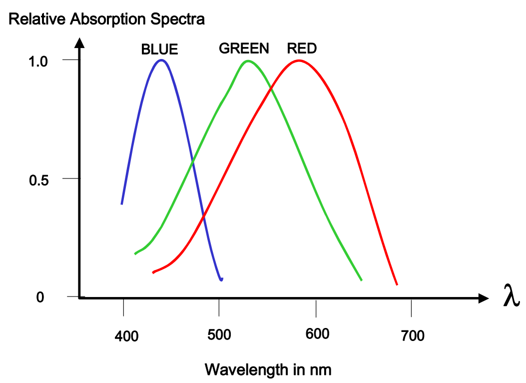

Figure 17 – Sensitivity of Colour Receptors

Flow of Light

The term *luminous flux* could be used in place of the term light, and although not in common usage, it does draw attention to some features of light that easily can be overlooked. Light is a flow of radiant energy, modified to take into account the propensity of the radiation to stimulate the sense of vision. It is the flow of light that is the stimulus of vision.

Detectability and Size

Objects < 6 minutes of arc - Ricco's law

$$\text{Detectability} = f(\text{Contrast} \times \text{Area})$$

2 degrees < Objects < 20 degrees - Pipers law

$$\text{Detectability} = f(\text{Contrast} \times \sqrt{\text{Area}})$$

Figure 18 – Detection of small optical fields

Where the light source is:

(i) small in size, (Ricco's Law, Piper's Law)
(ii) of short duration, (Bloch's law)

then other factors may need to be taken into account. The relationships in Figure 18 describe the factors that determine the detectability of small sized objects and in Figure 19 those that determine detectability of short duration flashes. In these circumstances it is the energy absorbed by the receptors that determines detectability.

Detectability and duration

For flashes of 0.2 seconds or more

$$E_e = \frac{E \times t}{a + t}$$

E = average illuminance at eye during flash
E_e = illuminance from an equally detectable steady signal
t = duration of flash in seconds
a = 0.2 seconds for achromatic light

For flashes shorter than 0.1 seconds

$$E_e = \frac{E \times t}{a}$$ Bloch's law

Figure 19 – Detection of brief flashes of light

However, this course is directed towards the transmission of natural light through facades and as such it will be concerned with steady sources of light at relatively high illuminances. Therefore it is the 'flow of light' that will determine the visibility of objects and be the subject of study.

There may be circumstances where the cumulative effects of light need to be considered. A minimum exposure to light in lx.hrs might be required in order to ensure either adequate growth in plants or to trigger particular developments in growth. In other situations, such as in an art gallery, a painting's total exposure of to light might need to be limited in order to prevent damage to the paint, canvas or frame.

Because of the additive nature of luminosity it is possible to sum the contributions from the different parts of the visual spectrum. If a determination is made of the relative degree to which each wavelength of radiation evokes a visual response, then the total potential for evoking a visual response is found by summing over all wavelengths the product of radiative power and sensitivity to radiation.

Visual Stimulus = Σ (power at λ) x (visual efficiency at λ)
where λ is the wavelength of the radiation.

Luminous efficiency of radiation

In principle the degree to which any wavelength of radiation evokes the sense of light can be found by undertaking the following experiment shown diagrammatically in Figure 20.

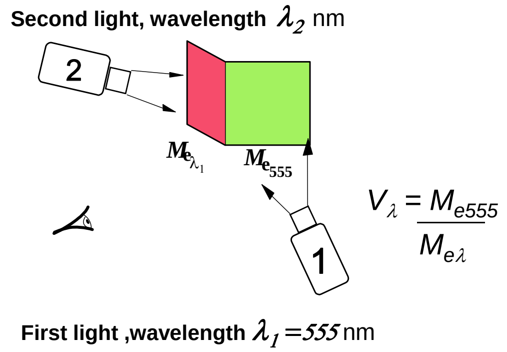

Figure 20 – Experiment to establish V_λ

Two radiation sources, S_1 and S_2 , are used. These can emit a variable power of radiation at various wavelengths. Keeping the power of S_1 constant, its wavelength is changed until the surface reflecting the light is seen to be at its maximum brightness. It will be found that this occurs when the light appears Green and has a wavelength of 555 nm.

The source S_2 is then set to a wavelength λ_2 that is slightly different to that of the first, 555nm. The power of S_2 is then altered until the second surface appears to be as bright the first surface.

The power at which this happens is recorded and the wavelength of the second source is then set to another wavelength and again the power altered until there is a match in brightness between the screen illuminated by S_2 and the screen illuminated by S_1. This process is repeated until the wavelength of S_2 no longer produces any visible effect, at which point the visual range of radiation will have been reached.

$$V_\lambda = \frac{M_{e_{555}}}{M_{e_\lambda}}$$

Figure 21 – Evaluating V_λ

From such an experiment it is found that the eye responds to radiation from about 380nm to 760nm. However the eye's sensitivity is very low at the extremes of the visual range and in these notes the visual range will be assumed to be from 400nm to 700nm.

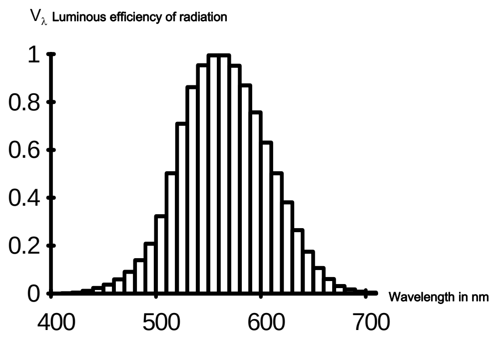

Figure 22 – V_λ Visual Efficiency of Radiation

The relative degree to which radiation evokes a visual response is called the *Luminous Efficiency of Radiation* and is usually denoted by the symbol V_λ. It may also be thought of as the relative sensitivity of the eye to radiation of different wavelengths.

From the experiment described above, if the radiant exitance of the surface lit with light of wavelength 555nm is $M_{e_{555}}$ watts/m^2, and this is matched in luminosity by the second surface lit with light of wavelength λnm to a radiant exitance of M_{e_λ} watts/m^2, then V_λ the luminous efficiency of radiation at a wavelength of λ nm is given by the relationship shown in Figure 21,

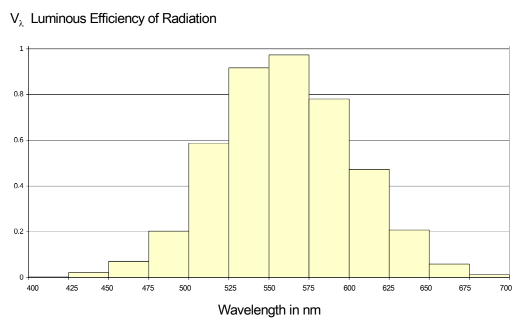

Figure 22 – Visual Efficiency of Radiation

The Luminous Efficiency of Radiation is shown plotted at waveband intervals of 10nm and 25nm respectively in Figures 21 and 22. The larger waveband of 25nm is used for sums in question sheets in order to reduce the amount of calculation.

Relative Visual Stimulus

For the radiation source shown in Figure 23, the potential visual stimulus is therefore going to be given by,

$$\text{Visual Stimulus} \propto \sum_{\lambda=400nm}^{\lambda=700nm} V_\lambda \, \Phi_{e_\lambda} \, \Delta\lambda \text{ light watts} \qquad \ldots 4$$

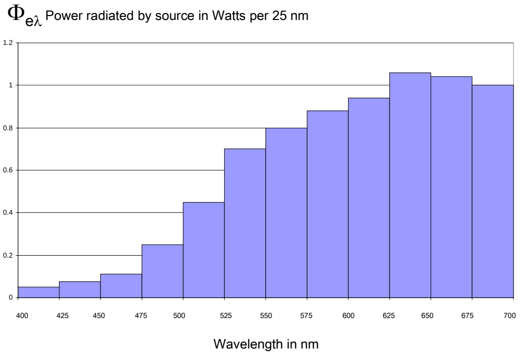

Figure 23 – Spectroradiometric curve of source

This summation is shown graphically in Figure 24. For each waveband in turn, the power of the source is multiplied by the visual efficiency to give the relative visual flux. The total relative visual flux will then be the area under the whole of the bottom graph.

The unit of light defined by reference to the relative ability of radiation to evoke the sense of light is described as a 'light watt'. This measure of light flux is not one that is commonly used and therefore it needs to be converted into the unit that has traditionally been used in lighting. This is the LUMEN and this is itself derived from the 'Standard Candle' that has previously been described.

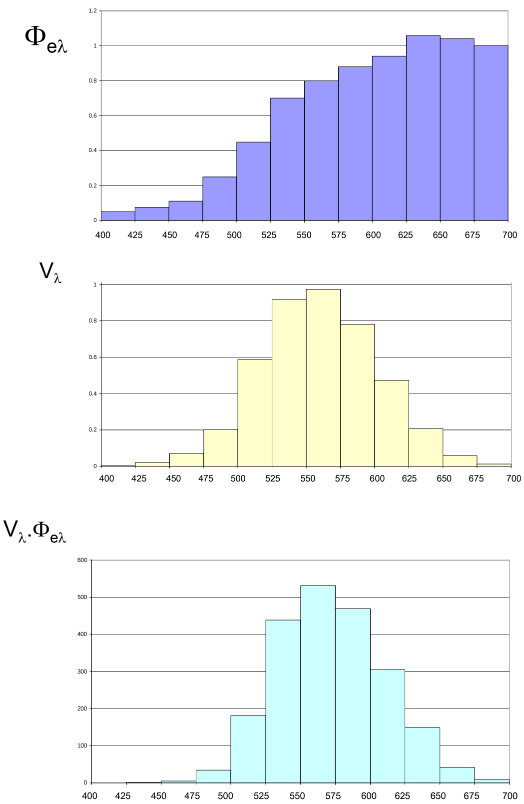

Figure 24 – Evaluating relative visual flux

The conversion of the relative measure of the 'Light watt' into the Lumen is accomplished by multiplying the relative measure by the absolute ability of radiation at a wavelength of 555nm to evoke the sense of light as measured in the original units of light flux, the lumen.

Luminous Efficacy of Radiation

The currently agreed maximum luminous efficacy of radiation occurs at a wavelength of 555nm and has a value of 683 lm/watt. It is generally given the symbol K_m.

Generally K_λ the luminous efficacy of a given wavelength of radiation would be defined as,

$$K_\lambda = \frac{\Phi_{V_\lambda}}{\Phi_{e_\lambda}} \text{ lm/watt}$$

Note - see next section for the definition of Φ_{v_λ}

The efficacy of a light source is defined differently because the loss of power by other modes of heat transfer needs to be taken into account as well as losses in ancillary equipment and control gear. Thus, where the total power consumed by the lamp and control gear is W watts, the efficacy of a lamp is given by,

$$\eta = \frac{\Phi_v}{W} \text{ lm/watt} \qquad \ldots 5$$

Light flux in LUMENS

The luminous flux measured in lumens is generally denoted by the symbol Φ_V and is given by,

$$\Phi_v = K_m \sum_{\lambda=400nm}^{\lambda=700nm} V_\lambda \Phi_{e_\lambda} \Delta\lambda \text{ lm} \qquad \ldots 6$$

and as $K_m = 683$ lm/w

$$\Phi_v = 683 \sum_{\lambda=400nm}^{\lambda=700nm} V_\lambda \Phi_{e_\lambda} \Delta\lambda \text{ lm} \qquad \ldots 7$$

The subscript 'v' is used to denote that the flux being described is visual flux or light, and in these notes this subscript will be omitted as being understood. Where it is necessary to make clear the difference between visual flux and flux of electromagnetic radiation, then the suffix 'e' will be appended to the emr flux.

Planar Illuminance

The higher the illuminance on a plane surface the better we can see details on that surface. The illuminance on a plane is the concentration of the incident light flux per unit area of plane surface and is known by the term *planar illuminance*. It is measured in *lux* and is defined as,

$$E_p = \lim_{\delta A \to 0} \frac{\delta \Phi_v}{\delta A} \quad \text{lux, lx (lm/m}^2) \qquad \ldots 8$$

$$E_p = \frac{\Phi}{A} \quad \text{lx}$$

Luminous Exitance

The concentration of light flux leaving a plane surface is known as the *luminous exitance* of the surface. It is measured in *apostilbs* and is defined as,

$$M = \lim_{\delta A \to 0} \frac{\delta \Phi_v}{\delta A} \quad \text{apostilb, asb (lm/m}^2) \qquad \ldots 9$$

$$M = \frac{\Phi}{A} \quad \text{asb}$$

The luminous exitance of an opaque surface is related to the illuminance by the surface reflectance, ρ,

$$M = E\rho \quad \text{asb} \qquad \ldots 10$$

These quantities are shown pictorially in Figure 25.

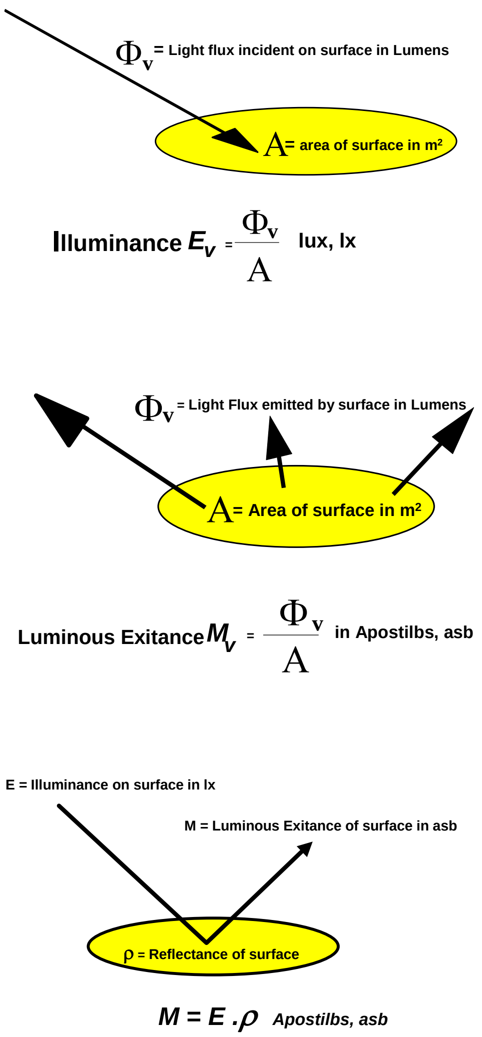

Figure 25 – Basic lighting units

Average illuminance

The approximate average illuminance in a room can be quite simply calculated if it is assumed that the light flux from all sources is evenly distributed over the surfaces of the room. Although this is a simplification of most real lighting installations it allows us to start to get a feeling for the quantities expected in real buildings. Also, for many circumstances it will provide an answer which is sufficiently close for the designer to check the feasibility of a design.

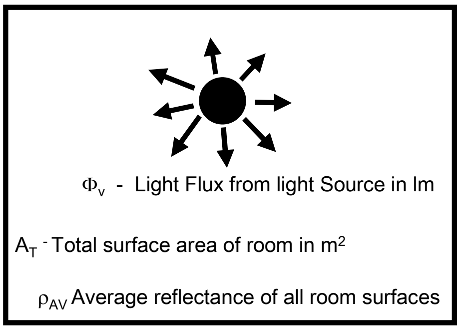

Figure 26 – Average illuminance in a room

$$E_{average} = \frac{\Phi_{source}}{A_T(1-\rho_{av})} \text{ lx}$$

where:

A_T = total surface area of room in m²,
$E_{average}$ = average illuminance over the surfaces in lux,
$\Phi_{sources}$ = light flux introduced into room in lumens
ρ_{av} = average reflectance of room surfaces.

By Conservation of Energy
light absorbed by room surfaces = light into room

But
light absorbed by room surfaces = (light flux incident on room surface) × absorbtance

$$= (E_{AV} \times A_T) \times \alpha$$

Therefore

$$E_{AV} \times A_T \times \alpha = \Phi_V$$

But that which is absorbed is that which is not reflected
and so, $\alpha = 1 - \rho_{AV}$

$$E_{AV} \times A_T \times (1 - \rho_{AV}) = \Phi_V$$

giving,

$$E_{AV} = \frac{\Phi_V}{A_T \times (1 - \rho_{AV})} \text{ lx}$$

Average reflectance

The average reflectance of the surfaces in the room must be an area weighted average.

$$\rho_{av} = \frac{\rho_1 A_1 + \rho_2 A_2 + \rho_3 A_3 + \rho_4 A_4}{A_1 + A_2 + A_3 + A_4}$$

where:

ρ_{av} = average reflectance of room surfaces,
ρ_n = reflectance of surface n,
A_n = area of surface n.

Illuminance at a single position

Introduction

Lumens
Illuminance
Luminous exitance

Luminous intensity
Luminance

Average illuminances and average daylight factors are both useful for outline design, predicting energy consumption and checking the feasibility of meeting design illuminances with given window sizes. These calculations of average illuminance and average daylight factor are possible using just three basic units of light introduced in the previous pages.

There are many occasions when the designer needs to know either the illuminance or daylight factor at a particular place in the room. In order to calculate the illuminance at some specified position, two additional lighting quantities need to be introduced.

These new lighting quantities are **luminous intensity** and **luminance**. Both of these rely upon the fiction of a point source of light. Such a light source proves to be very useful, and although a fiction it is closely approximated by many sources of light.

Before introducing these two additional lighting units, there is needed some additional preliminary information. Particularly there is a need to be familiar with the measure of solid angle.

Plane Angle

A plane angle between two intersecting lines is a measure of the part of a plane between those lines. The angle included between two straight lines can be described either in terms of '*degrees*' or '*radians*'.

The unit of a degree results from dividing a plane around a point into 360 equal parts. The angle included between two intersecting lines is then the number of those 1/360th parts that will fit between the two lines, as indicated in Figure 26.

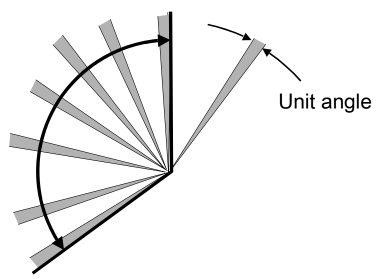

Figure 26

The choice of dividing the plane into 360 parts has no sound basis other than it has a long history. A less arbitrary measure of plane angle is the radian.

The radian is based upon the fact that the length of the arc of a circle centred on the intersection of the two lines will be proportional the part of the plane included between the two lines, as is shown in Figure 27. The length of the arc is also proportional to the radius of the circle. Therefore in order to provide a measure that is independent of the radius of the circle

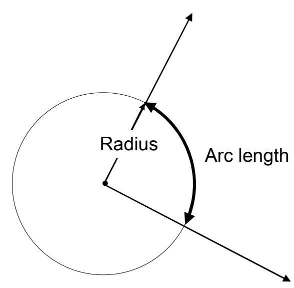

Figure 27

drawn, the arc length needs to be divided by the radius. This results in a measure of angle that is called the radian.

$$\alpha = \frac{l}{r} \quad \text{radians}$$

where

α angle in radians

l length of arc in m.

r radius of circle in m.

If the whole plane about a point is considered, then the total number of radians about a point will be equal to the circumference of the circle divided by its radius. Thus there are 2π radians about a point.

Solid angle

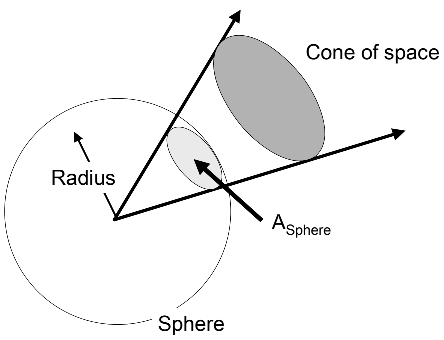

Figure 28

Solid angle is a measure of the volume of space within an envelope of space that originates from a point as is illustrated in Figure 28. It may be directly compared to plane angle measured in radians, except that the measure describes a part of three dimensional volume rather than a part of a two dimensional plane.

The space included within a cone that radiates from a point is proportional to the area of intersection on a sphere that is centred on the origin of the cone. This area on the sphere is also proportional to the radius², and therefore to make the measure independent of the radius, the area on the sphere must be divided by the radius². This gives a measure of solid angle in Steradians. The symbol used to denote solid angle is usually ω, and the units Steradians are contracted to sr.

$$\omega = \frac{A_{Sp}}{r^2} \quad \text{sr}$$

where

ω solid angle in steradians

A_{Sp} area on sphere in m^2

r radius of sphere in m.

The total solid angle of space about a point will therefore be the area of a sphere divided by the radius², giving a total of 4π Steradians about a point.

Projected Area

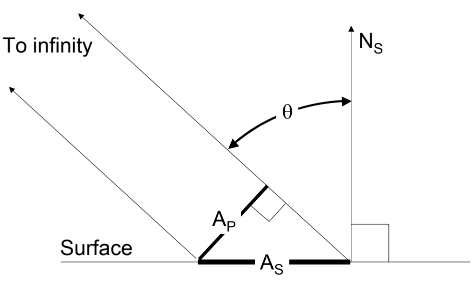

Figure 29

A plane that is seen at an oblique angle will appear smaller than when seen head on. If the plane is viewed from infinity then the area of the plane when seen at an oblique angle is known as the Projected Area of the plane. This is shown in Figure 29 where it can be seen that:

$A_P = A_S \cos\theta$

where

A_P projected area in m^2.

A_S surface area in m^2.

θ angle of view from normal

Approximation to solid angle

The solid angle is not always an easily computed quantity and therefore when considering finite areas it is found that it is often simpler to use an approximation for the solid angle. This approximation is,

$$\frac{A_P}{d^2} \approx \frac{A_{Sp}}{r^2} = \omega \quad \text{sr.}$$

The diagram in Figure 30 shows a cross section through a spherical cap. The projected area of the cap is the disc formed by the edge of the spherical cap.

The area of the spherical cap is shown derived at the margin and by reference to Figure 31, and is equal to,

.

$A_{Cap} = 2\pi R\ (1-\cos\alpha)\ m^2$

The solid angle subtended by the cap is therefore given by:

$$\omega = \frac{A_{Cap}}{R^2} = \frac{2\pi R^2(1-\cos\alpha_1)}{R^2} = 2\pi(1-\cos\alpha_1) \quad \text{sr.}$$

By reference to Figure 31 the projected area of the cap can be seen to be given by the disc area:

$$A_P = \pi r^2 = \pi R^2 \sin^2\alpha$$

and AP/d² will therefore be:

$$\frac{A_P}{d^2} = \frac{\pi R^2 \sin^2\alpha}{d^2}$$

$$but\ as\ d = R\cos\alpha$$

$$\frac{A_P}{d^2} = \frac{\pi R^2 \sin^2\alpha}{R^2\cos^2\alpha} = \pi\tan^2\alpha$$

Comparing the approximate values of solid angle to the true values in Table 1, it can be seen that even for fairly large subtended angles the error is quite small.

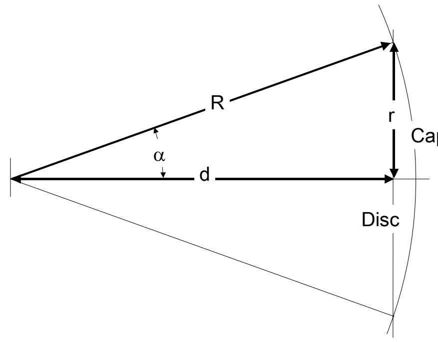

Figure 30

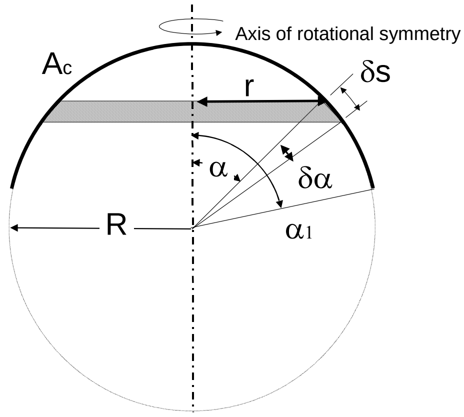

Figure 31

Area of elemental ring on sphere

$\delta A = \text{width} \times \text{circumference}$

$\quad = dS \times 2\pi r$

$\quad = R\,d\alpha \times 2\pi\, R\sin\alpha$

$\delta A = 2\pi\, R^2 \sin\alpha\, d\alpha$

And area of a spherical cap will be,

$$A_c = 2\pi R^2 \int_{\alpha}^{\alpha_1} \sin\alpha\, d\alpha$$

$$= 2\pi R^2 \left[-\cos\alpha\right]_{\alpha}^{\alpha_1}$$

$$= 2\pi R^2 (\cos\alpha_0 - \cos\alpha_1)$$

$$= 2\pi R^2 (1-\cos\alpha_1)$$

α in deg.	α in rad.	ω in sr.	A_P/d^2	% error
0.1	0.001745	0.0000096	0.0000096	0
0.5	0.00873	0.0002392	0.0002393	0
1	0.01745	0.00096	0.00096	0
5	0.08727	0.02391	0.02405	+0.59
10	0.1745	0.09546	0.09768	+2.3
20	0.3490	0.37892	0.41618	+9.8
30	0.5235	0.84179	1.04720	+24
45	0.78540	1.84030	3.14159	+70

Table 1 Error incurred in using an approximation for solid angle

Luminous Intensity

The luminous intensity of a point source of light may be considered in either of two ways.

1) Luminous intensity is the illuminating power in a particular direction of a point source of light. As such it is the constant of proportionality between the illuminance and the inverse of the distance squared.

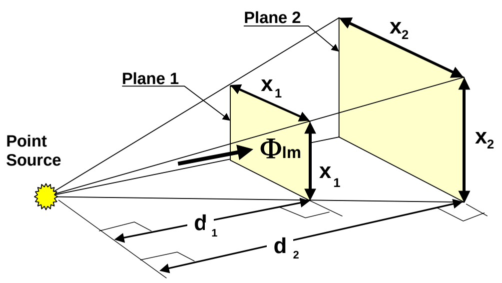

Figure 32 – Point Light source

Consider the diagram in Figure 32 that shows light flux emanating from a point source. This light illuminates two planes placed perpendicularly to the flow of light. As light travels in straight lines, all the light flux passing through the first plane will also pass through the second plane. Thus the light flux incident on the two planes is the same. Therefore the normal illuminances incident on the two planes will be given by,

$$E_{N_1} = \frac{\Phi}{A_1} \text{ and } E_{N_2} = \frac{\Phi}{A_2} \qquad \ldots 11$$

and because the light flux incident on both is the same,

$$\frac{E_{N_1}}{E_{N_2}} = \frac{A_2}{A_1} \qquad \ldots 12$$

However from the diagram it can be seen that the ratio of areas

$$\frac{A_2}{A_1} = \frac{x_2 \times x_2}{x_1 \times x_1} \qquad \ldots 13$$

And also from the diagram, by similar triangles,

$$\frac{x_2}{x_1} = \frac{d_2}{d_1} \text{ and therefore } \left(\frac{x_2}{x_1}\right)^2 = \left(\frac{d_2}{d_1}\right)^2 \qquad \ldots 14$$

which when substituted into 13 gives,

$$\frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2$$

and when this used in the relationship given in 12,

$$\frac{E_{N_1}}{E_{N_2}} = \left(\frac{d_2}{d_1}\right)^2 .$$

This can be re-arranged as,

$$E_{N_1}\left(d_1\right)^2 = E_{N_2}\left(d_2\right)^2 = \text{ constant}$$

This constant is given the name ***luminous intensity***, and is represented by the symbol ***I*** ,

$$E_N = \frac{I}{d^2} \quad \text{lx} \qquad \ldots 15$$

The unit of Intensity is the ***Candela***, ***cd***, and a light source of luminous intensity 1 *cd* will produce an illuminance of 1 lx on a perpendicular surface 1m from light source.

The second definition of Intensity is,

2) The *Luminous Intensity* of a point light source is the concentration of luminous flux in a particular direction.

Figure 33 shows a point source of light where the light flux is emitted within a small cone of space. The *luminous intensity* of the point source of light in the direction of the cone may be defined as,

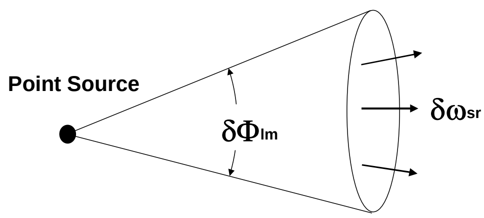

Figure 33 – Luminous Intensity

$$I = \lim_{\delta\omega \to 0} \frac{\delta\Phi}{\delta\omega} \quad \text{candela , cd} \qquad \ldots 16$$

where, $\delta\Phi$ = light flux in lumens, lm
$\delta\omega$ = solid angle in steradians, sr

Illuminance on a plane from a point source

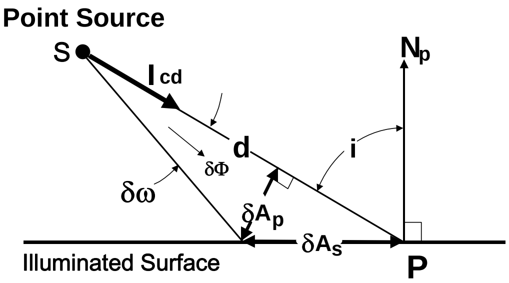

Figure 34 – Illuminance from a point source

Figure 34 shows a point light source *S*, with a luminous intensity *I* cd in the direction of P on the surface. Consider the light falling over δA_S the elemental area on the surface at *P* .

The light incident over the small elemental surface area δA_S will pass through δA_p , the projected area of δA_S in the direction of the light source. The true area on the surface and its projected area are related by the angle of incidence *i* ,

$$\cos i = \frac{\delta A_p}{\delta A_S},$$

and therefore, $\delta A_p = \delta A_S \cos i$ …17.

By definition the solid angle subtended by the elemental area on the surface of the plane is ,

$$\delta\omega = \frac{\delta A_p}{d^2},$$

which using $\delta A_p = \delta A_S \cos i$ from 17 gives

$$\delta\omega = \frac{\delta A_S \cos i}{d^2} \qquad \text{…18}$$

The light flux $\delta\Phi$ contained within the solid angle $\delta\omega$ can be found directly by re-arrangement of the definition of *luminous Intensity* , so that

$$\delta\Phi = I\ \delta\omega,$$

and substituting from 18 for $\delta\omega$ gives,

$$\delta\Phi = I\frac{\delta A_S \cos i}{d^2} \qquad \text{…19}$$

The *illuminance* at *P* is, $E = \frac{\delta\Phi}{\delta A_S}$, and substituting from 19 for $\delta\Phi$, the δA_S's cancel, leaving the point source formula for *illuminance* as,

$$E = \frac{I\cos i}{d^2}\ \text{lx} \qquad \text{…20}$$

Luminous Intensity

For a real source the luminous intensity is established quite simply by measuring the illuminance on a surface at some known distance from the source as is shown schematically in Figure 35.

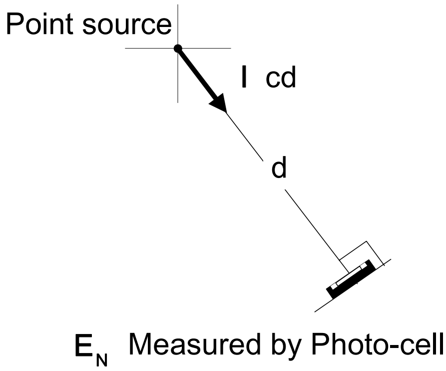

Figure 35 measuring luminous intensity

Re-arranging the point source formula gives:

$$I = \frac{Ed^2}{\cos i} \text{ cd.}$$

If the illuminance is measured normal to the light direction, then the angle of incidence i will be 0 and cos(i) = 1. If E_N denotes the illuminance normal to the direction of light, then the luminous intensity may then be found by applying the relationship:

$$I = E_N d^2 \quad \text{cd.}$$

Luminous Intensity is a vector quantity as it has magnitude and direction. It is possible to represent the intensity of a point source of light by a solid, where the intensity in a particular direction is related to the distance of the surface from the centre in a particular direction, as is shown in Figure 36.

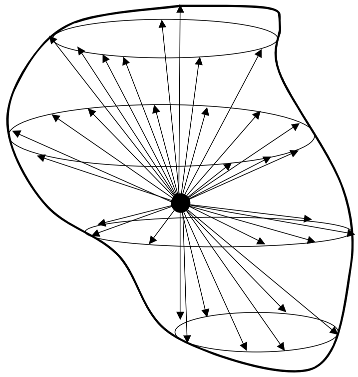

Figure 36 – Solid of luminous intensity

It is not easy to represent complex solids and it is fortunate that many light sources are rotationally symmetric about an axis as is shown in Figure 37.

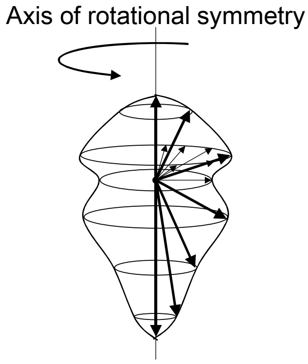

Figure 37 – Symmetrical distribution

Where this is so, then the luminous intensity can be more simply represented by the Polar intensity diagram shown in Figure 38, known as a Polar Curve of luminous Intensity. The polar solid can be formed from such a curve by rotating the polar curve through 360 about the axis of rotational symmetry.

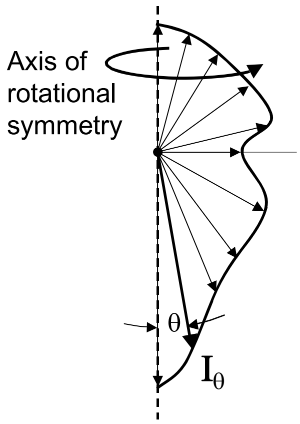

Figure 38 – Polar Curve of Intensity

Light Flux emitted by point sources

Establishing the total light flux emitted by a point source of light is a practical problem faced by luminaire manufacturers. It is not a difficult theoretical problem, as can be seen by what follows.

The luminous intensity in a particular direction may be determined as described above, by measuring the normal illuminance at some distance from the source.

An equation for the light flux within an elemental solid angle can be found by re-arranging the definition of Luminous Intensity given in equation 16:

$$\delta\Phi = I\delta\omega \quad \text{lm}$$

The total light flux from a source may then be found by integrating this quantity over all solid angles,

$$\Phi = \int d\Phi = \int I.d\omega \ . \ \text{lm}$$

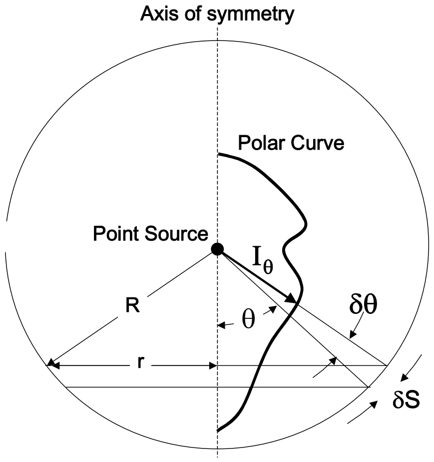

Figure 39 – Flux from Polar curve

Figure 39 shows a sphere surrounding a point light source with an elemental ring of width δS drawn on the sphere at an angle of θ° to the axis of rotational symmetry. The solid angle subtended by the elemental ring is,

$$\delta\omega = \frac{\delta \mathrm{A}_{\mathrm{Sphere}}}{\mathrm{R}^2}$$

the area of the ring will equal its circumference by its width, and so the solid angle of the elemental ring will be,

$$\delta\omega = \frac{2\pi r \times \delta s}{R^2} \quad \text{sr}$$

and as $r = R\sin\theta$ and $\delta S = R\delta\theta$,

$$\delta\omega = \frac{2\pi R\sin\theta R\delta\theta}{R^2} = 2\pi\sin\theta \ \delta\theta \ \text{ sr.}$$

The flux emitted within the elemental ring will therefore be,

$$\delta\Phi = 2\pi I_\theta \sin\theta \ \delta\theta \quad \text{lm}$$

and for the whole source,

$$\Phi = 2\pi \int_{\theta=0}^{\theta=\pi} I_\theta \sin\theta \ d\theta \quad \text{lm.}$$

The intensity I_θ in a direction θ may generally be described in terms of a function of the intensity at some reference angle, say when θ = 0. So that for instance,

$$I_\theta = I_0 f(\theta) \quad \text{cd.}$$

Most luminaires have intensity distributions where the $f(\theta)$ is not a simple trigonometrical function. When this is the case the integration needs to be undertaken using one of a number of different graphical or numerical methods. These methods will not be considered in this course.

If a light source does have a distribution that can be described by a simple trigonometrical function then the total light flux it emits will be given by:

$$\Phi = 2\pi I_0 \int_{\theta=0}^{\theta=\pi} f(\theta)\sin\theta\, d\theta \quad \text{lm.}$$

And this may be evaluated by analytical integration.

Luminance

Luminance is perhaps the most confusing of the lighting units. This is perhaps not helped by the quantity having such a similar name to illuminance and also because it is used incorrectly in a number of text books where it is confused with the luminous exitance of a surface. Just to remind ourselves, luminous exitance *M* asb, refers to the concentration per unit area of all the light flux exiting a surface.

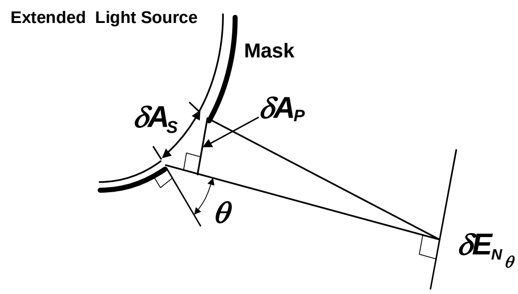

Figure 40 – Illuminance from extended source

Luminance may be thought of as the illuminating power of an extended light source in a particular direction. Bearing in mind that the definition of the illuminating power of a point source is Luminous Intensity, then it is not too much of a step to consider the luminance as the intensity per projected area of a light source. Using L_θ as the symbol representing the luminance in the direction θ,

L_θ = illuminating power / projected area of source

$$L_\theta = \frac{I_\theta}{A_p} \text{ candela/m}^2\text{, cd/m}^2$$

The luminance of a surface in a particular direction is determined in a manner similar to that used to find the intensity of a point source. Consider Figure 40 where a small area of a finite sized source is left unobscurred and the illuminance on a surface normal to the flow of light is measured far away from the source of light. This is done so that the subtended area of the source is small and it can be considered as a point source of light. The luminance will be given by,

$$L_\theta = \frac{\delta I_\theta}{\delta A_P} \qquad \text{…21}$$

and from measurement the intensity at angle θ will be given by,

$$\delta I_{\theta} = \delta E_{N_{\theta}} d^{2}$$

which when substituted for the intensity in 21 gives

$$L_{\theta} = \frac{\delta E_{N_{\theta}} d^{2}}{\delta A_{P}} \quad \text{cd/m}^2$$

and as $\delta A_P/d^2$ is the solid angle subtended by the source at illuminated plane, the luminance may defined as,

$$L_{\theta} = \lim_{\delta\omega \to 0} \frac{\delta E_{N_{\theta}}}{\delta\omega} \qquad \ldots 22$$

It might be noted that the illuminance itself is defined as $\delta\phi/\delta A$, therefore one formal definition is in the terms,

$$L_{\theta} = \frac{\delta^{2}\Phi}{\delta\omega\delta A} \qquad \ldots 23$$

Illuminance from finite or extended light sources

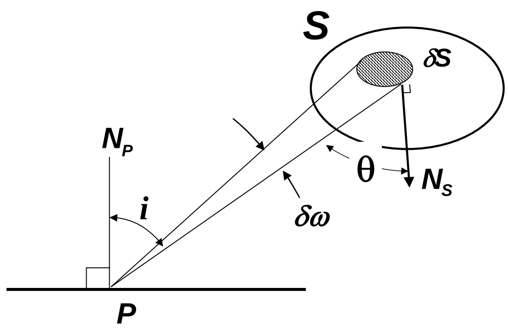

Figure 41 – Illuminance from finite source

Re-arranging the formal definition of luminance given in 22, the illuminance on a surface normal to the light is given by,

$$\delta E_{N_{\theta}} = L_{\theta}\delta\omega$$

and the illuminance on an oblique plane is related to the normal illuminance by,

$$E_{p} = E_{N} \cos i$$

Therefore the illuminance on plane at P from element of source as shown in Figure 41 is,

$$\delta E_{P_{\theta}} = L_{\theta} \cos i \,\delta\omega$$

and from the whole source it will be,

$$E_{P} = \int_{S} L_{\theta} \cos i \, d\omega \qquad \ldots 24$$

For the particular case of a surface which has a uniform diffuse luminance where L_{θ}= const. = L,

$$E_{P} = L \int_{S} \cos i \, d\omega \qquad \ldots 25$$