7 PREDICTING THE SUN'S POSITION

7.1 Introduction

There are many different ways of representing the position of the sun in the sky and most of these may be usefully used to help investigate sunlighting in design. In order to limit explanations, only one or two of the methods will be considered in any detail, but once the principles are understood, one should find it easy to apply those to other methods not described in detail.

The basic astronomical facts will be reviewed but a detailed knowledge of them is not essential for an appreciation of sunlighting.

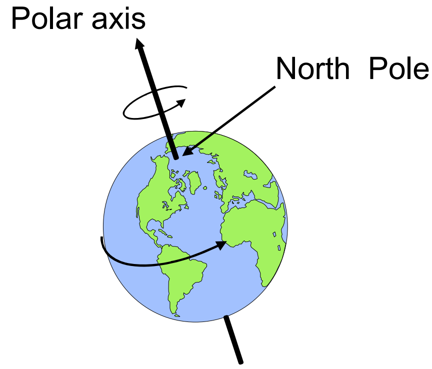


Figure 7.1 – Earth's rotation

7.2 The Earth

The Earth is effectively a spherical globe that rotates eastwards about a North-South axis approximately once every 24 hours, as shown in Figure 7.1.

A globe may be partitioned in various ways as shown in Figure 7.2. These prove useful in locating accurately different places on the Earth.

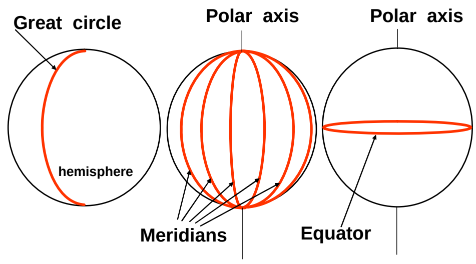


Figure 7.2 –A sphere

If a globe is divided into two equal parts to produce two hemispheres, then the dividing line between the two parts will be a ***Great Circle***.

The axis about which the Earth rotates is known as the Polar Axis and this axis intersects the globe at the North Pole and the South Pole. A great circle passing through both poles is known as a ***Meridian***.

A great circle that is equidistant from the North and South Poles is known as the ***Equator***.

As shown in Figure 7.3, any place on the Earth may be specified in relation to;

i) a primary meridian,
ii) the equator.

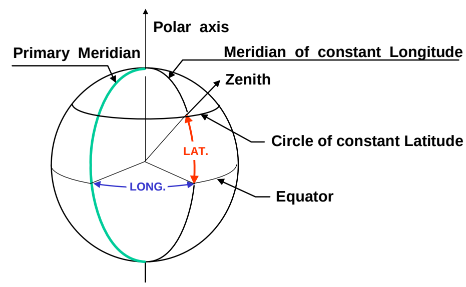


Figure 7.3 – Latitude and Longitude

Longitude describes the position of the appropriate meridian in relation to the primary meridian. It is measured around the equator in degrees East or West of the primary meridian.

Latitude describes the angle in degrees from the equator towards a Pole along a particular meridian – it may be either North or South of the equator.

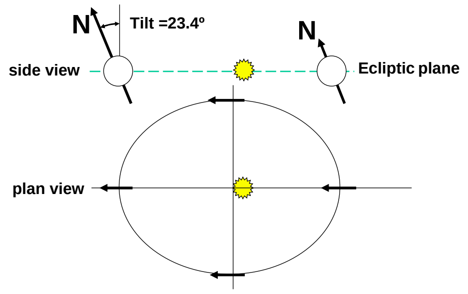


Figure 7.4 – Earth's orbit

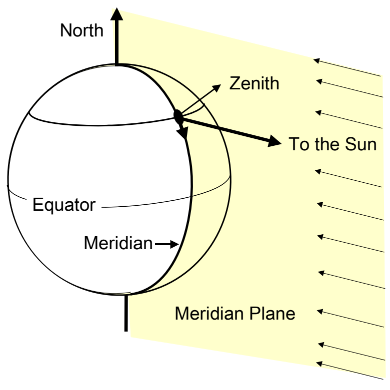


Figure 7.5 – Sun at noon

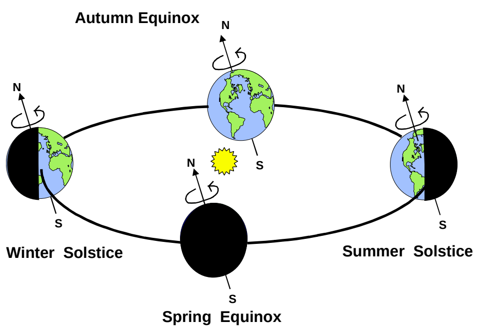


Figure 7.6 – Constant tilt

7.3 The Earth's orbit around the Sun

The Earth orbits the Sun approximately once every 365 days. Its orbit lies in the same plane as the Sun and is elliptical in shape with the Sun positioned at one of the ellipse's foci as shown in Figure 7.4. This plane is known as the ***Ecliptic Plane*** because when the moon moves into the plane there is the possibility of an eclipse.

One consequence of the elliptical orbit is that the earth speeds up and slows down as it moves around the sun and this means that the length of the day, measured from noon to noon, changes slightly throughout the year.

Noon at a particular location in the northern latitudes occurs when the sun is due south. This occurs when the sun passes through the Meridian plane formed by the polar axis and the location on the earth as is shown in Figure 7.5.

In northern latitudes the sun appears to be due South at noon whilst in southern latitudes the sun appears due North at noon.

The changing length of the ***Solar Day*** rather complicates time keeping and it is convenient to assume a constant length of day and use the average length of day throughout the year. This leads to the familiar time convention in the UK of Greenwich Mean Time, which is based upon the mean or average length of day over the whole year.

The difference between solar time and local mean time is called the ***equation of time***. A correction should be applied for the equation of time when it is required to know the position of the sun very accurately. However, the maximum cumulative difference between local solar and mean time is in the order of between +15 minutes and -15 minutes and for many architectural purposes it may be ignored.

Because the earth rotates eastwards, local solar noon is also dependent upon the Longitude of a location. In the UK this is usually ignored as the correction is only 4 minutes for each degree change in Longitude.

Daylight saving corrections – British Summer Time - are applied.

7.3.1 The tilt of the Earth's axis and orbital plane

A most important feature of the Earth's circumstance is that the Polar Axis is tilted in relation to the orbital plane as shown in Figure 7.4 and Figure 7.6. Within the time spans considered in architecture the direction of the Polar axis relative to the orbital plane remains constant. At the present time the Tilt is at an angle of 23.4º.

One consequence of the tilted axis is that seasons of the year are experienced by those parts of the globe closer to the poles. The closer a region is to one of the poles, then the more seasonal is the climate experienced by that region.

Arctic circles and tropics

The tilt also gives rise to the division of the globe into various parts shown in Figure 7.7.

The Arctic Circles divide the regions of the Earth into those that will at some time in the year experience a 24 hour day and a 24 hour night, and those regions where the sun will cross the horizon.

The Tropics divide the regions of the Earth into those where the sun will be directly overhead at some time in the year and those where the sun will never reach the Zenith.

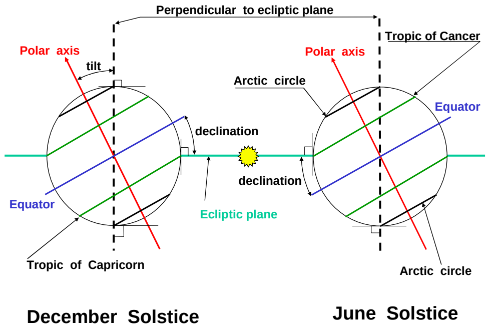


Figure 7.7 –Tropic and Arctic

Declination

The tilt of the earth's axis results in a change in the relative position of the sun as the earth moves in its orbit. This change in the relative position of the sun is reflected in the change that occurs in the angle the sun's rays make with the equatorial plane. This angle is known as the ***declination*** , and is shown schematically in Figure 7.8.

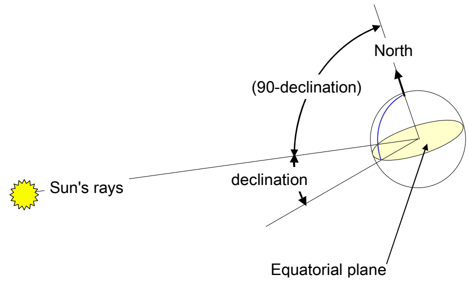


Figure 7.8 – Declination

The declination will vary from a maximum of 23·4º at the Summer Solstice to a minimum of -23·4º at the Winter Solstice. Twice in the year the declination will be zero and this occurs at the Spring and Autumn Equinox.

There are thus 4 times in the year when the declination takes particular values that are especially significant:

Astronomical occurance	D	Calendar date
Winter Solstice	-23.4	23rd. December
Vernal (Spring) Equinox	0	21st. March
Summer Solstice	+23.4	21rd. June
Autumnal Equinox	0	23rd. September

The Winter Solstice is that time of year when the declination is a minimum. Because the direction of Tilt is not exactly in line with major axis, it is not the case that this coincides with the shortest day of the year.

The Summer Solstice is that time of year when the declination is a maximum. Similarly to the other solstice, the longest day does not necessarily coincide with the summer solstice.

The Equinoxes are those times of year when the day and night are of equal time. Thus the sun will rise at 6am and set at 6pm solar time.

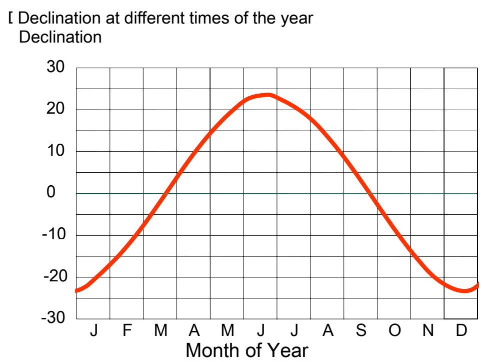


Figure 7.9 Declination Values

At other times in the year the Declination may be read from a chart such as is shown in Figure 7.9 or found using:

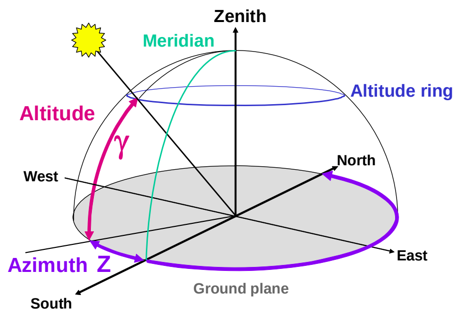


Figure 7.10 – Hemisphere of sky

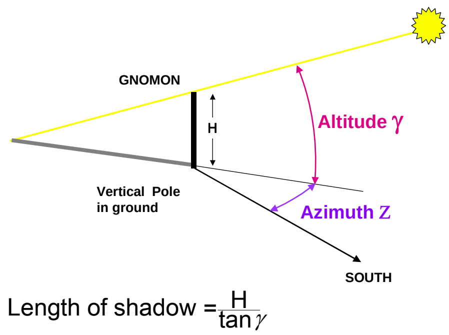
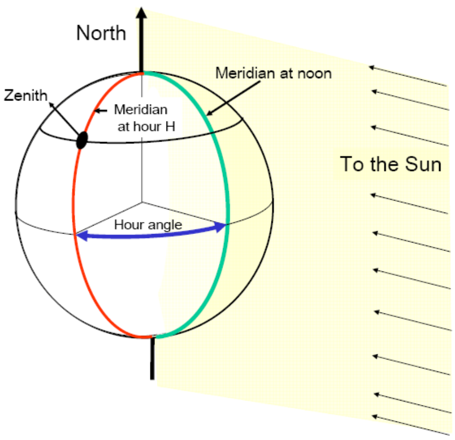


Figure 7.11 Shadow of a pole



Symbol	Variable	Definition
D	Declination	The angle of the sun's rays to the equatorial plane, positive in summer.
L	Latitude	The angle from the equator to a position on Earth's surface.
H	Hour angle	The angle the Earth needs to rotate to bring the meridian to noon. Each hour of time is equivalent to 15 deg.
N	Day Number	The day number, January 1st is 1.

Figure 7.12 Definitions

$$Declination = 23.4 \times \sin\left(\frac{360 \times (284 + N)}{365}\right) \quad \text{degrees}$$

Where N is the day number of the date for which the declination is being calculated. January 1st. being day number 1.

There are more accurate formulas and one is given in the course titled 'Lighting and acoustics' that may be found on my web site.

7.4 Positioning the sun in the sky

The position of the sun in the sky is described by two angles that are shown in Figure 7.10,

γ - Greek symbol gamma - the ALTITUDE of the sun above the ground (horizon) plane,

z – the AZIMUTH, which is the compass direction of the sun on the ground plane.

The Azimuth may be quoted in two ways; either degrees clockwise from North or alternatively, degrees East of or West of South. In general, the azimuth is most often given in terms of the angle from North, but in these notes the angle will be given as an angle in degrees East or West of South.

Figure 7.11 shows a shadow cast by a vertical pole and how the length and position of the shadow are determined by the altitude and azimuth of the sun.

The sun's position of altitude and azimuth may be determined from the following equations:

$$\sin\gamma = \cos D \times \cos L \times \cos H + \sin D \times \sin L\,,$$

$$\cos z = \frac{\cos D \times \cos H \times \sin L - \sin D \times \cos L}{\cos\gamma}\,,$$

$$\sin z = \frac{\sin H \times \cos D}{\cos\gamma}\,,$$

$$\tan z = \frac{\sin H}{\sin L \times \cos H - \cos L \times \tan D}\,.$$

These equations are NOT to be learnt, however they can be useful when you need to position the sun accurately. The hour angle H, is defined in Figure 7.12

The above formulae are not the most convenient way of considering the sun's position and a graphical approach may be more useful.

7.4.1 Sundials

The *Gnomon* is a point in space through which the rays of the sun pass to later shine upon some surface, as shown in Figure 7.13. The shadow on the ground cast by a flagpole will depend upon the sun's altitude and azimuth, and if the shadow of the topmost tip of the flagpole is considered, then it will sweep out a path on the ground as the sun moves across the sky. The topmost tip of the flagpole may be considered as a gnomon.

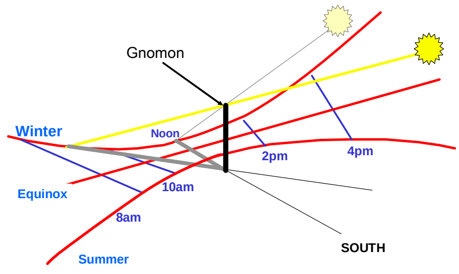


Figure 7.13 – Sun dial

It may occur to you that a simple perspective is also constructed as a gnomic projection.

Plotting the paths of the tip of the shadow for different times throughout the year will produce a sun dial as shown in Figure 7.13.

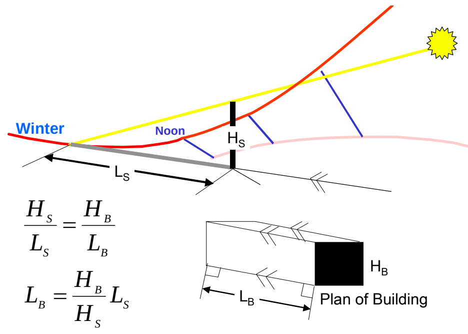


Figure 7.14 –drawing shadows

Using the horizontal sundial

If the height of the gnomon is known then the sundial can be used to construct the shadows created by buildings at different times of year. This is done simply by using the fact the ratio of the shadow length L_S with gnomon height of the sundial H_S will be exactly the same as the ration of the length of a building's shadow L_B with building's height H_B. The direction of the shadow will be the same as that on the sundial. This is shown diagrammatically in Figure 7.14.

Sun patches created by sunlight shining through windows can be constructed in a similar manner to show the effect of sunlight through different types of window at different times of year as in Figure 7.15.

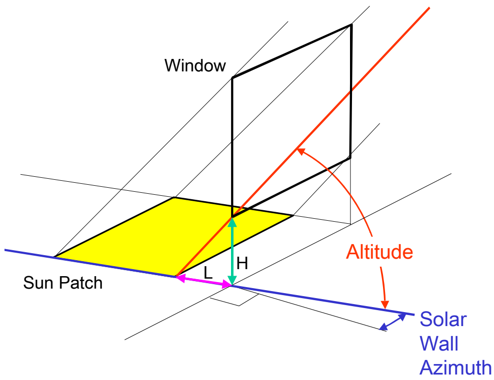


Figure 15 – Drawing sun patches

The sundial may also be used in what is sometimes called the aviator's method. If the sundial is placed on a model and is viewed so that the gnomon is lined up with a particular time of year, then the eye is positioned in the direction of the sun at that time of year as in Figure 716. All that the eye will see on the model will therefore be exposed to sunlight at that time of year. Clearly, as the eye is very much nearer to the model than the sun, there will be increasing discrepancies as ones gaze moves away from the centre of the sundial. For this reason it is best to view the model from some distance away so that ones direction of view stays fairly constant as one views the whole model.

Sundials do not need to be on horizontal surfaces but can also be drawn on vertical surfaces. They are of course only correct for that one orientation of wall.

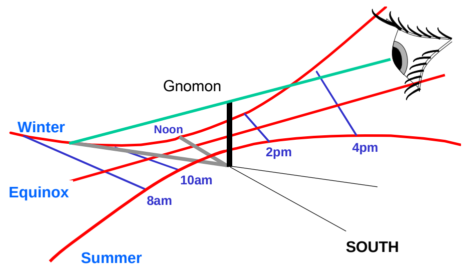


Figure 7.16 – Aviator's method

However, vertical sundials can be used with perspective drawings because a perspective is a gnomic projection. This sort of sunpath diagram is the basis of those constructed for the book 'Windows and Environment' edited by Turner and produced by Pilkington's.

7.4.2 Sunpath diagrams

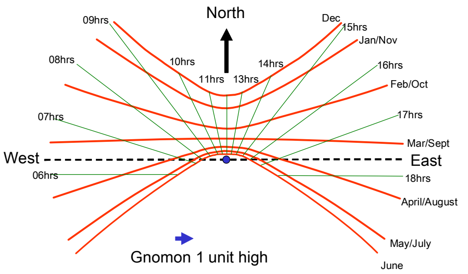


Figure 7.17 – Horizontal sundial

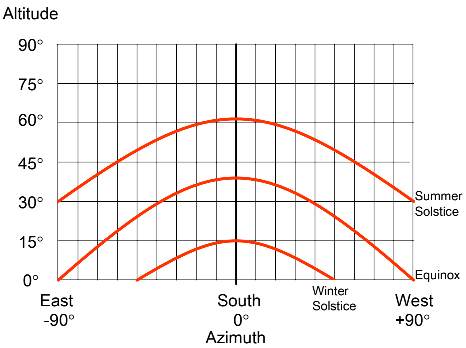


Figure 7.18 Rectangular diagram

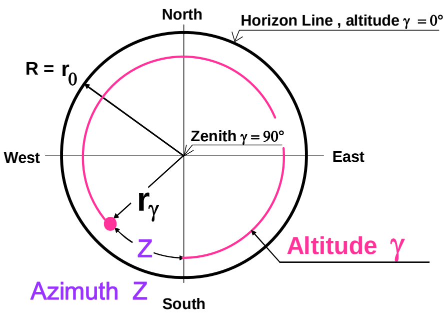


Figure 7.19 Radial diagram

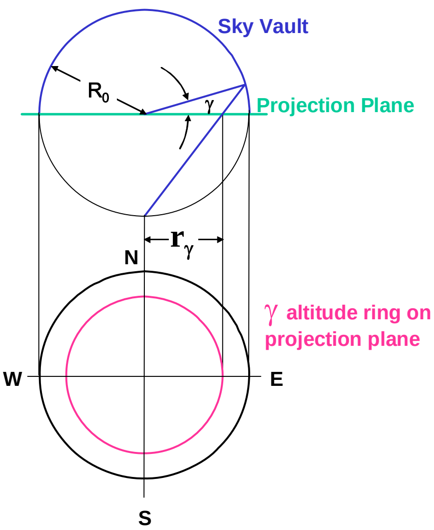


Figure 7.20 Stereographic proj.

There are many different ways of graphically displaying the relative position of the sun at different times of the day and year. These range from,

i) Sun dials, these are gnomic projections, see Figure 7.17,
ii) Rectangular projections of the sky, see Figure 7.18,
iii) Radial or Circular sunpath diagrams, see Figure 7.19.

There is no one diagram that is preferable to all others, because each has its own advantages and disadvantages. I believe that the Stereographic form of the radial diagram has distinct advantages as it can be easily sketched by hand because all the sunpaths and time lines are arcs of circles, and these are reasonably easy to sketch for even the non artistic. Also, a radial diagram allows the whole sky to be considered in one diagram and so there is no need to construct a new diagram for each different orientation of the façade as is required for rectangular diagrams and gnomic projections on a vertical plane.

However its advantages are less pronounced with the advent of the computer, as now calculations and redrawing can be undertaken without effort. Never the less, it still is a most useful way of considering the sun's position. In computer applications, it does lend itself to serious design because it allows the consideration of a number of variables at one time, and this is not always possible with some of the other techniques of displaying sunpaths.

The Stereographic projection

The basis of the circular projections is that a hemisphere of sky is projected down onto a horizontal plane.

This results in a diagram of the form shown in Figure 7.19, where the points of the compass are defined by the *direction* out from the centre of the diagram, and the altitude is defined by the *distance* out from the centre.

The construction of the Stereographic projection is shown in Figure 7.20 and this results in the relation between radius and altitude,

$$r_\gamma = R_0 \tan\left(\frac{90° - \gamma}{2}\right)$$

7.4.3 Sun's position in the sky

When *sketching* a sunpath for a particular location, it is really only necessary to consider four times in the year,

i) the Winter Solstice,
ii) the Summer Solstice,
iii) the Spring (Vernal) and Autumnal Equinoxes.

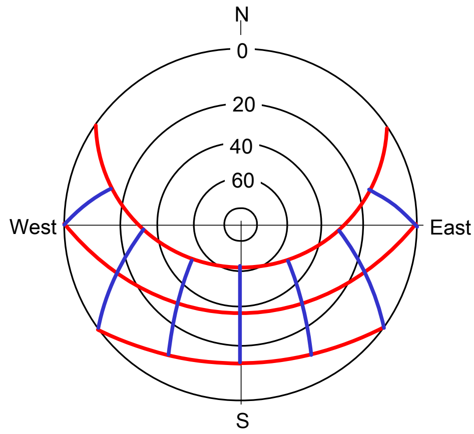


Figure 7.21 Sunpath for Bath

The two Equinoxes have the same declination of 0° and so have the same sun path loci. Therefore for most design purposes no more than three sun path loci need be plotted on the sun path diagram as shown in Figure 7.21. In this diagram the three red lines are the sunpaths for the Winter solstice, the Summer Solstice and the Equinoxes. The blue lines are hour lines at two hourly intervals with the vertical line being solar noon.

On the stereographic projection the sunpath loci and hour lines are arcs of circles and the arc of a circle can be sketched quite easily if three positions on an arc are identified. This is shown in Figure 7.22.

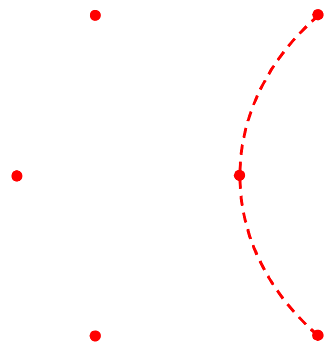


Figure 7.22 Arcs of circles

The three positions in the sky used to sketch a Sunpath for a given day are:

i) the altitude of sun at solar noon,
ii) the azimuth of sun rise,
iii) the azimuth of sun set.

Because the declination of the sun changes but a little during the course of a day, a single value of declination may be used for any one day. Therefore the azimuths of sun rise and sun set may be taken to be symmetrically located on either side of south. Thus only 2 sums need to be undertaken for any one sunpath loci.

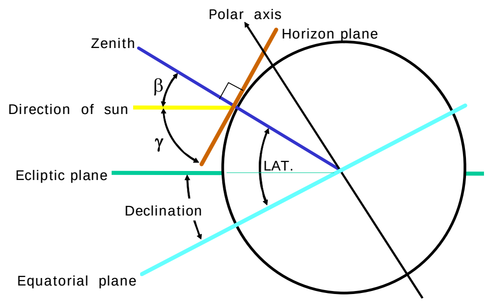


Figure 7.23 – Altitude at Noon on summer solstice

Calculating the altitude at solar noon

At Latitudes between the Tropics and the Arctic circles

Noon occurs when a location's meridian is closest to the sun as shown in Figure 7.5. An enlarged section along the meridian is shown in Figure 7.23 and this may be used to show the altitude of the sun at solar noon is given by:

$$\gamma_{Noon} = 90° - L + D .$$

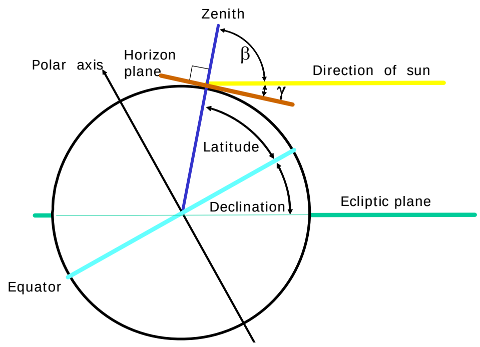


Figure 7.24 – Altitude at Noon on winter solstice

Note that by convention the declination in Summer is positive and the declination in Winter is negative. Therefore if you work out the altitude of the sun at noon in Winter from Figure 7.24, you will find that the altitude is again given by:

$$\gamma_{Noon} = 90° - L + D$$

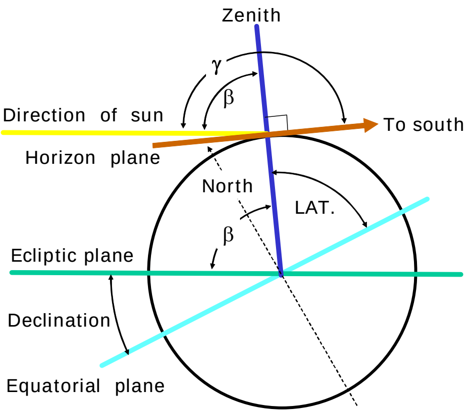


Figure 7.25 – Altitude at Midnight

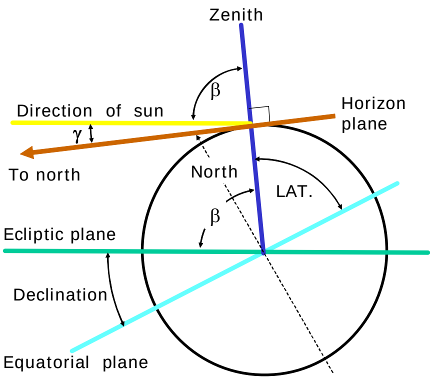


Figure 7.26 – Altitude at Midnight

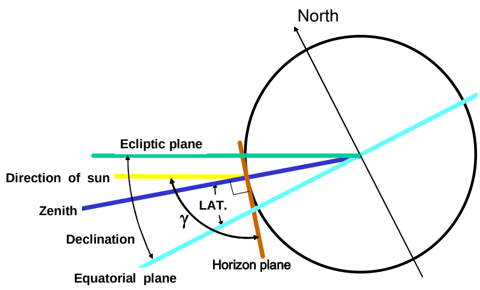


Figure 7.27 – In the Tropics

At Latitudes above the Arctic circles

At latitudes above the arctic circle the sun is above the horizon for the whole day during the summer solstice. This means that on a stereographic projection the sun path locus will be a circle. All that is needed to draw a circle is its diameter. One end of the diameter will be given by the position of the sun a noon and this can be found using the formula previously given. The other end of the diameter will be the sun's position at midnight. If the altitude of the sun is measured from the southern horizon as is the altitude at noon, then Figure 7.25 can be used to show the value is given by the formula,

$$\gamma_{Midnight} = 270° - L - D \,.$$

If the altitude of the sun is measured from the horizon in the direction of the sun, which is north, then from Figure 7.26 the altitude will be given by the formula:

$$\begin{aligned}\gamma_{Midnight} &= 180° - (270° - L - D) \\ &= L + D - 90°\end{aligned} \,.$$

More important than trying to remember conventions is to try and picture what is happening on the diagram and to compare this with your knowledge of what happens in reality at different latitudes.

At Latitudes in the Tropics

The aspect that can cause confusion is the same as that just encountered when describing where the sun is at midnight in the arctic. The altitude may be measured always from the same direction, for northern latitudes, say south, as shown in Figure 7.27, where the altitude is given by the same formula as previously used:

$$\gamma_{Noon} = 90° - L + D \,,$$

and the altitude will be greater than 90° and pass over the zenith.

Or when the sun passes over the Zenith the altitude may be measured from the Horizon in the direction from which the sun is seen i.e. north when the altitude will be:

$$\gamma_{Noon} = 90° - D + L$$

Calculating the azimuth of sunrise and sunset

The azimuth of the sun at sun rise and sun set may be found using the relationship given below.

$$\cos z_{\gamma=0} = \frac{-\sin D}{\cos L} \,.$$

It is worthwhile noting that, where the sun does not cross the horizon, the above relation does not hold. Therefore, there will be no solution when the latitude is >66° for either the summer, or the winter solstice.

7.4.4 Sketching a Sunpath for Bath

As an example, the process of sketching a sunpath diagram for the City of Bath will be worked through in some detail. Note that the Latitude of Bath is 51·3° North.

Sunpath for the Spring and Autumn Equinoxes

Position of the sun at solar noon

Note that at solar noon the sun is due South and the maximum altitude is given by the relation:

$$\gamma_{Noon} = 90° - L + D\,.$$

Using the declination of D = 0° for the spring equinox on March 21st. and autumnal equinox on September 23rd., the altitude of the sun at solar noon will be given by:

$$\begin{aligned}\gamma_{Noon} &= 90° - 51.3 \\ &= 38.7°\end{aligned}$$

This is plotted on the stereographic projection in Figure 7.28.

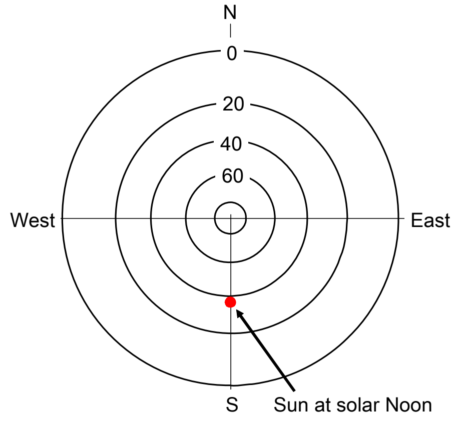


Figure 7.28 –Solar Noon on Equ.

Position of the sun at sunrise and sunset

Note that at the equinoxes the sun is above the horizon and below the horizon for the same length of time. Therefore the length of day is the same as the length of night. Also the sun rises due (directly) East and sets due West. When the sun rises, its altitude is 0° and therefore the sun position lies on the outer circle of the stereographic projection. The sunrise and sunset positions are plotted on Figure 7.29.

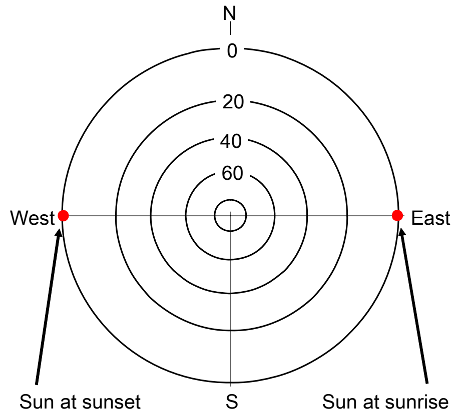


Figure 7.29 –Sunrise and sunset

There are now plotted three positions on the Sunpath locus for the equinox, and these may be used to sketch the first of the Sunpath loci, as is shown in Figure 7.30.

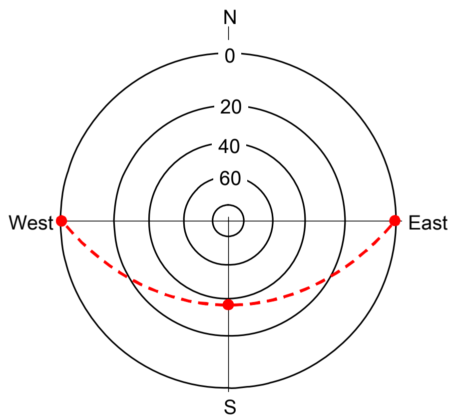


Figure 7.30 –Sunpath at Equinox

Sunpath for the Summer Solstice

Position of the sun at solar noon

For the summer solstice, June 21st., when the declination is +23·4°,

$$\begin{aligned}\gamma_{Noon} = 90° - L + D &= 38.7° + 23.4° \\ &= 62.1°\end{aligned}$$

As you have already calculated the altitude of the sun at the equinox, the value for (90° - L) i.e. 38.7° can be used to make the sum easier. The sun is due south at noon and the sun's position is shown plotted on Figure 7.31

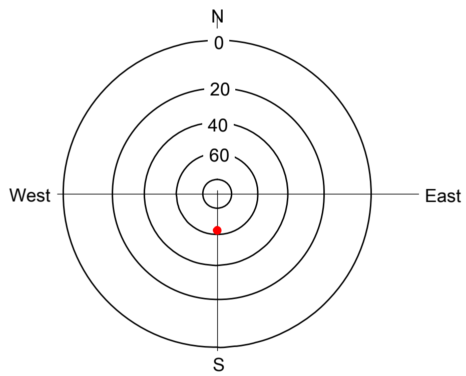


Figure 7.31 –Noon at Sum. Sols.

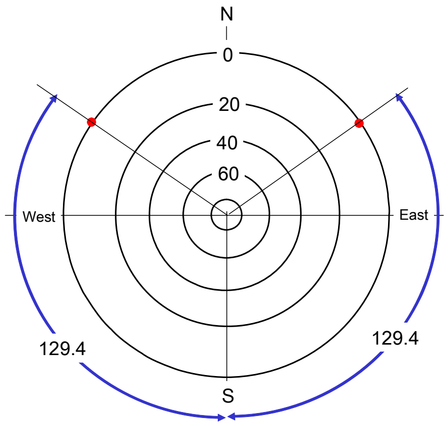


Figure 7.32 Position at $\gamma=0$

Position of the sun at sunrise and sunset

Use the relation given for the azimuth at sun rise and sunset,

$$\cos z_{\gamma=0} = \frac{-\sin D}{\cos L}$$

The declination at the summer solstice is +23.4° and therefore,

$$\cos z_{\gamma=0} = \frac{-\sin 23.4°}{\cos 51.3°} = -\frac{0.397}{0.625} = -0.635\,.$$

Taking the inverse of the cosine, the azimuth when sun is at an altitude of 0°, i.e. on the horizon is,

$$z_{\gamma=0} = \cos^{-1}(-0.635) = 129.4°$$

Therefore the azimuth of sunrise and sunset are 129·4° East and 129.4° West of South, as is shown in Figure 7.32.

These three positions of the sun can be connected together by an arc of a circle that gives the Sunpath for the summer solstice as shown in Figure 7.33.

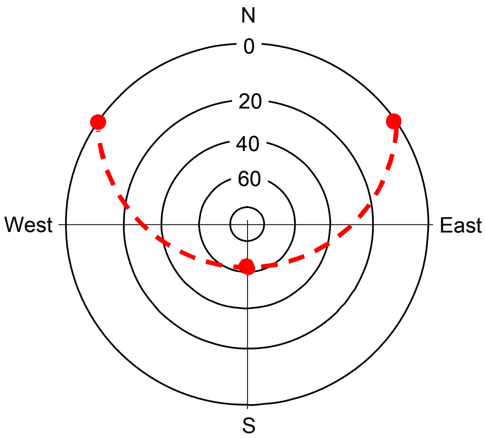


Figure 7.33 Summer Sunpath

Sunpath for the Winter Solstice

Position of the sun at solar noon

For the winter solstice, December 23rd., when declination is –23·4°,

$$\gamma_{Noon} = 38.7° - 23.4$$
$$= 15.3°$$

And this is plotted on Figure 7.34.

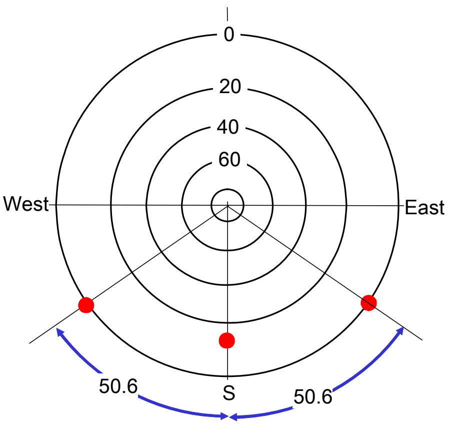


Figure 7.34 Positions in winter

Position of the sun at the equinoxes

The declination is –23·4° and therefore using the relation given for the azimuth at sunrise and sunset,

$$\cos z_{\gamma=0} = \frac{-\sin(-23.4°)}{\cos 51.3°} = \frac{+\sin 23.4°}{\cos 51.3°} = \frac{0.397}{0.625} = 0.635$$

And taking the inverse function

$$z_{\gamma=0} = \cos^{-1}(0.635) = 50.6°\,.$$

The two positions east and west are plotted on Figure 7.34 and the three points are connected together as a sunpath in Figure 7.35.

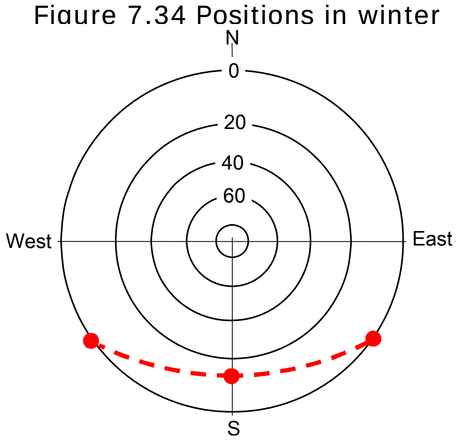


Figure 7.35 Sunpath in winter

In order to clarify the previous calculation, it should be noted that using the trigonometrical identity:

$$.\sin(A-B)=\sin A\cos B-\sin B\cos A$$

Then if $A=0°$, $\sin A=0$ & $\cos B=1$,

And therefore $\sin(-B)=-\sin B$.

Sketching lines representing solar hours

The Sunpath loci for the three times of year are shown on the same diagram in Figure 7.36.

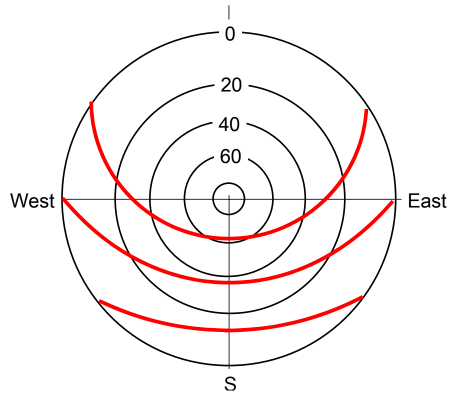


Figure 7.36 – Sun paths

The sunpath diagram will be completed by adding the arcs of circles that represent different hours of the day. All the time lines represent solar time.

The method used to sketch the time lines is not exact, but given that there are inaccuracies that result from not applying the equation of time and considering the use of the diagram, the errors introduced are inconsequential.

The initial stage is to locate three certain and accurate times. These are:

i) Solar noon occurs when the sun is due South,
ii) At the Equinoxes the sun rises due East at 6am,
iii) At the Equinoxes the sun sets due West at 6pm.

These time positions are located on the diagram shown in Figure 7.37.

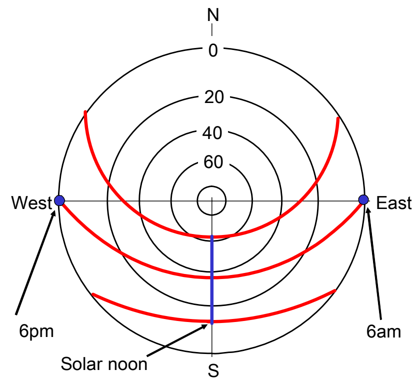


Figure 7.37 – Time positions

It then just needs to be noted that the time lines are arcs of circles and that they cross the sunpath loci at right angles i.e. 90°.

Figure 7.38 shows the hour lines for 6am and 6pm sketched onto the diagram.

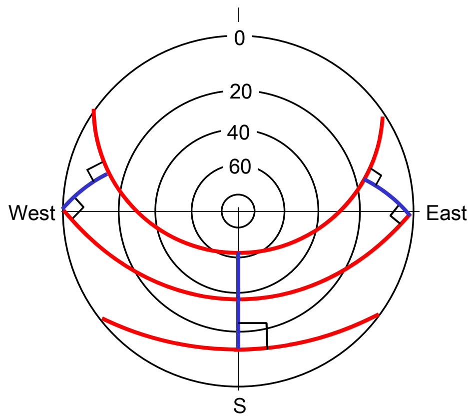


Figure 7.38 – Hour Lines

The intermediate hour lines may be positioned on the basis of spacing them equally along the equinox sunpath between the noon and 6 o' clock hour lines. There is little reason to plot time lines at intervals less than two hours as this might imply a greater level of precision than is warranted. However, the resulting diagram shown in Figure 7.39 is extremely useful in giving the designer an idea of where is the sun at different times of day and year.

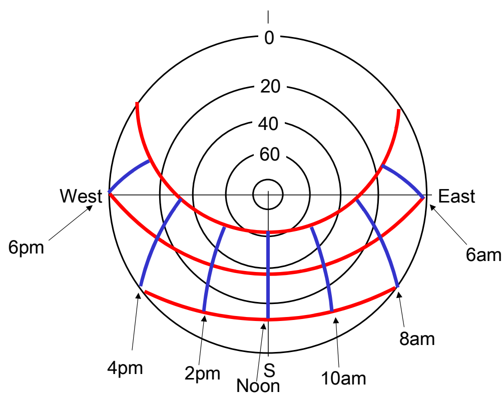


Figure 7.39 – Sunpaths for Bath

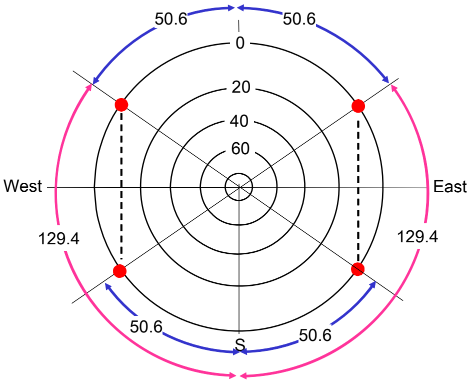


Figure 7.40 Azimuth Symmetry

Short cut

Using the trigonometrical identity:

$$\cos(180° - z) = \cos 180° \cos z + \sin 180° \sin z$$

Then as $\cos(180°) = -1 \; and \; \sin(180°) = 0$

$$\cos(180° - z) = -\cos z \, .$$

If the azimuths of sunrise for the Summer Solstice and the Winter Solstice are reviewed, it will be seen that they are respectively 129·4° and 50·6°. These add up to 180°, as would be expected from the above result because the modulus of the declination is the same with only a change in the sign.

This means that there is no need to go through the calculation of the azimuth for the summer and winter solstices. It is simpler do the azimuth calculation for the winter solstice and then to use the fact that the sunrise and sunset for the two Solstices are symmetrically positioned about the East-West axis. This is shown diagrammatically in Figure 7.40.

7.4.5 For Southern Latitudes

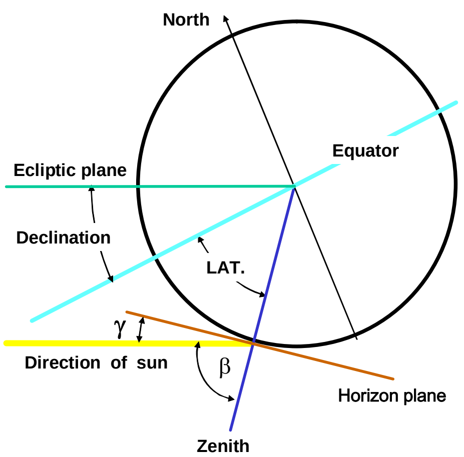


Figure 7.41 Winter, D is -tve

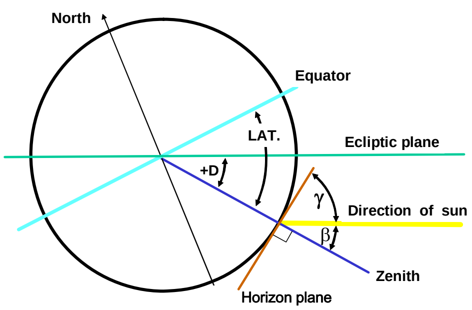


Figure 7.42 Summer, D is +tve

In southern latitudes the sun will primarily be due north at solar noon. The sun will still rise in the East and set in the West. If there is doubt in your mind about where the sun is, then consider again a diagram of the cross section through the ecliptic plane as in Figure 7.41 and Figure 7.42.

There are a number of different conventions that can be applied to the construction of diagrams for southern latitudes. However, the simplest approach is to always use a positive declination for summer and a negative declination for winter. In southern latitudes summer occurs in December and winter in June. Altitudes should be measured from the north horizon and if greater than 90° then the sun will move across the zenith into the southern hemisphere of sky, i.e. just the opposite to the northern latitudes.

If all else fails then just draw the sunpath diagram for a northern latitude and then rotate the diagram by 180°, i.e. just turn it upside down. Just be careful to remember that summer occurs in December in southern latitudes and winter occurs in June.

7.5 The Stereographic projection

The stereographic projection displays the whole sky on a flat plane. In order to fully utilise the diagram, it is necessary to relate it to three dimensional reality, and if you only look at the diagram as a two dimensional drawing then you will be lead into confusion.

Particularly useful in conjunction with the projection itself is a shading protractor such as is shown in Figure 7.43. The upper arcs divide the hemisphere of sky into a series of inclined planes.

A single inclined plane is shown in Figure 7.44, and it can be seen that it divides the hemisphere of sky into two parts; that above the inclined plane and that below the inclined plane.

This inclined plane is positioned within the hemisphere of sky by two quantities:

i) The Vertical Shading Angle – VSA,
ii) The orientation of the Ground Line –GL.

Figure 7.45 shows the single inclined plane of Figure 7.44 plotted on a stereographic projection. It is worth noting that :

i) the altitude of the inclined plane (VSA), and the altitude circle of VSA coincide on a line normal to the Ground Line,

ii) the altitude of the inclined plane is zero where the Ground Line crosses the horizon line,

iii) the locus is an arc of a circle on the stereographic projection.

These facts mean that if you do not have a protractor you can quite easily sketch one by identifying the three positions of (i) & (ii) and then using (iii) to sketch an arc between the three points.

A special case of the inclined plane is when the VSA is 90°. Such a vertical plane is shown in Figure 7.46, and Figure 7.47 shows this plane plotted on a stereographic projection. Being simply a vertical plane passing through the centre of the projection, it has a constant azimuth and will just be a radial line emanating from the centre of the projection.

The protractor of Figure 7.43 has a series of radial lines representing vertical planes on one side of the ground line.

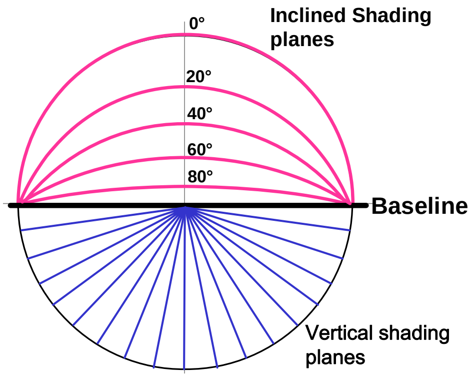


Figure 7.43 Shading protractor

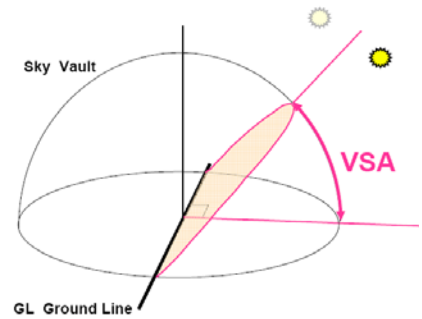


Figure 7.44 Inclined plane

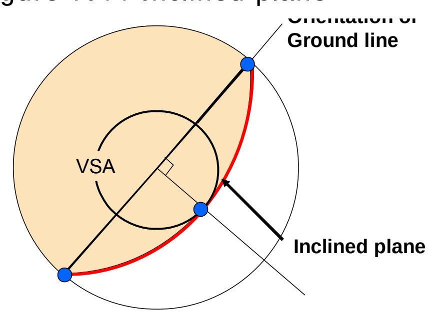


Figure 7.45 Inclined plane

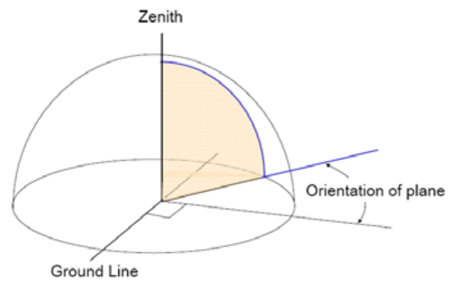


Figure 7.46 Vertical plane

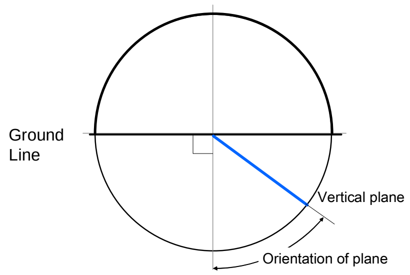
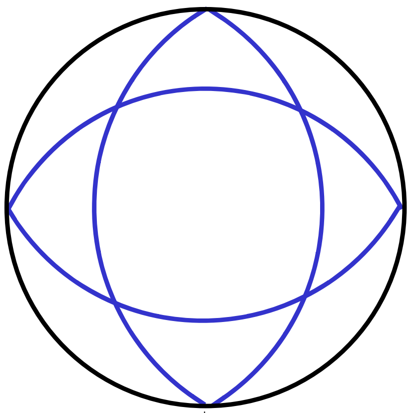
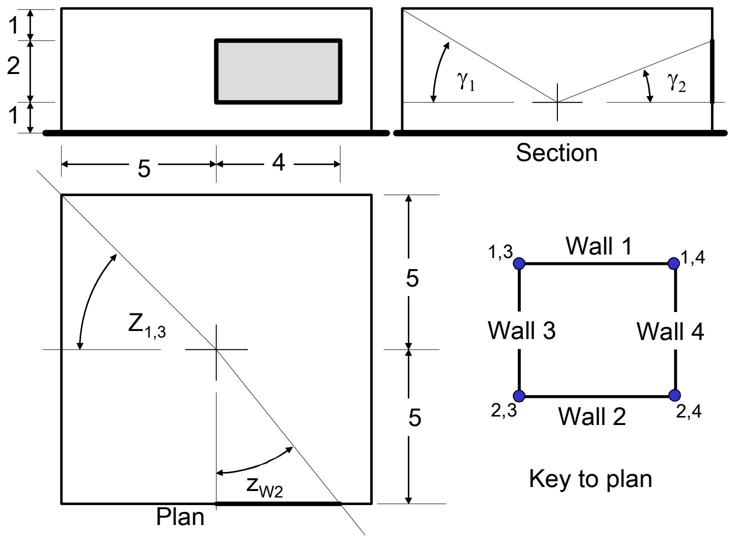
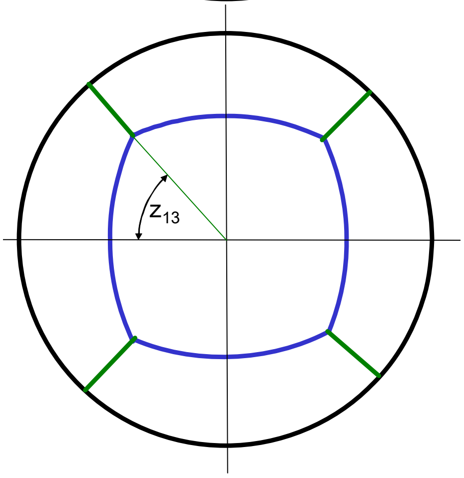


Figure 7.47 Vertical plane





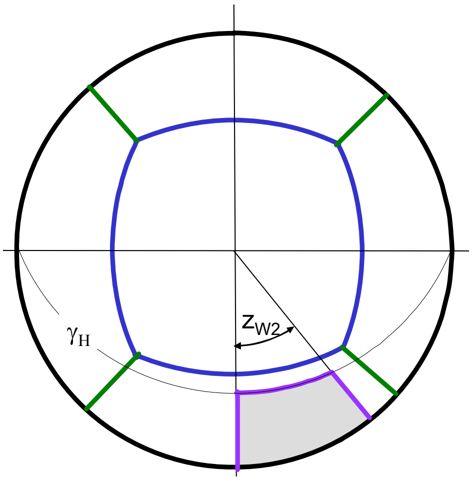
Viewing a room



The stereographic projection may be used to display a room as seen from some point. Consider the 10m square room shown above in plan and section. Assume that a projection is required, centred on the middle of the room at a height of 1m. Each of the wall's intersection with the ceiling will lie in an inclined plane emanating from the middle of the room. The inclined planes, each with their own vertical shading angle γ_N , will therefore represent the junction of the ceiling and walls. For each wall;

$$\gamma = \tan^{-1}\frac{3}{5} = 31°$$

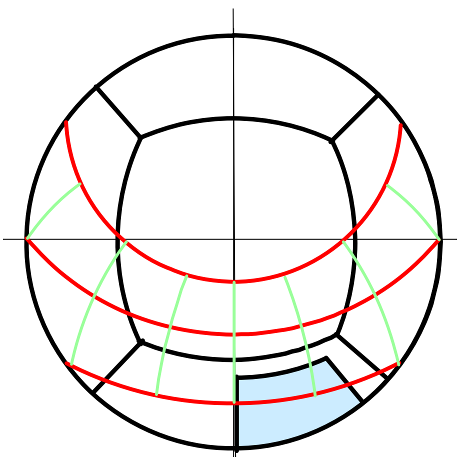
These are plotted on a stereographic projection with the shading protractor and using the angle of VSA=31°.



The walls intersect in a vertical plane with a constant azimuth and therefore are shown by radial lines. These may be constructed either by using the angle $Z_{1,3}$, or drawn as a radial lines emanating from the corners of the ceiling.

The vertical sides of the window subtend an azimuth of Z_{W2} and the head of the window lies in an inclined plane of angle γ_H where;

$$\gamma_H = \tan^{-1}\frac{2}{5} = 21.8° \text{ and } Z_{W2} = \tan^{-1}\frac{4}{5} = 38.7°$$



Thus one can draw a stereographic projection of the room seen from its centre. This projection may then be superimposed over a Sunpath diagram and the sunpaths seen through the aperture of the window will be seen by the point at the centre of the room. Thus the hours different positions receive sunlight can be investigated by drawing stereographic projections of the rooms from the points of interest and superimposing a sunpath diagram over the drawings.

Clearly the room projection should be correctly orientated with respect to the Sunpath diagram.